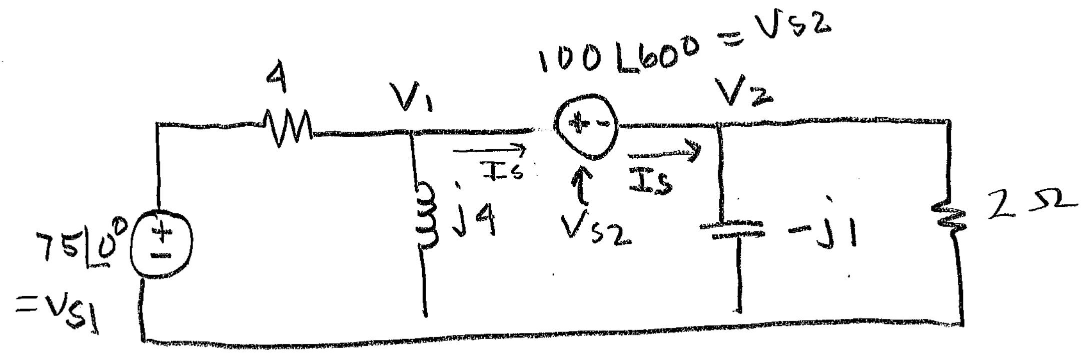
LECTURE 9 AC STEADY STATE

Practice problem 10.2 Pg 417



Find V_1 and V_2

$$\text{①} \quad \frac{V_1 - V_{s1}}{4} + \frac{V_1}{j4} + I_s = 0$$

$$\text{②} \quad -I_s + \frac{V_2}{-j1} + \frac{V_2}{2} = 0$$

$$\text{③} \quad V_1 = V_2 + V_{s2}$$

3 EQ, 3 UNK V_1, V_2, I_s

Use "supernode" (Pg 417) if you wish

$$\text{①} \quad V_1\left[\frac{1}{4} + \frac{1}{j4}\right] + 0\,V_2 + I_s = \frac{V_{s1}}{4}$$

$$\text{②} \quad 0\,V_1 + V_2\left[\frac{1}{-j1} + \frac{1}{2}\right] - I_s = 0$$

$$\text{③} \quad V_1 - V_2 + 0\,I_s = V_{s2}$$

$V_{S1} = 75\angle 0^\circ = 75 + j0$

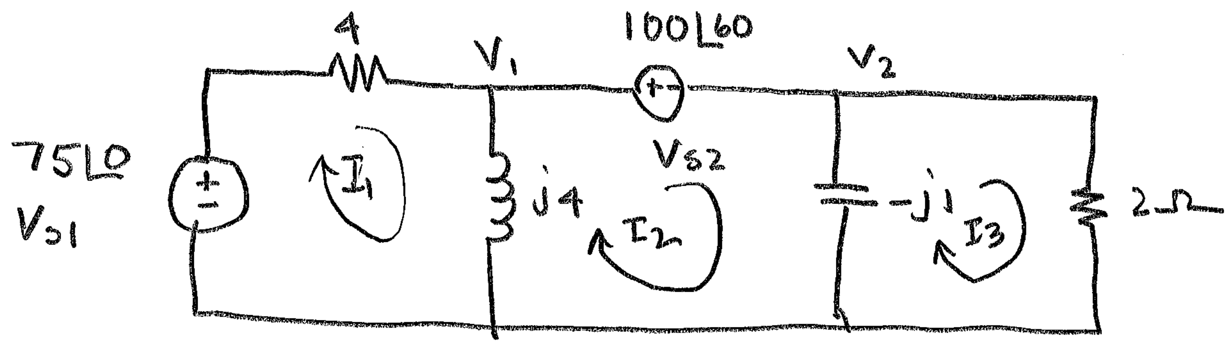
$V_{S2} = 100\angle 60^\circ = 50 + j86.6$

$$\begin{bmatrix} \frac{1}{4}+\frac{1}{j4} & 0 & 1 \\ 0 & \frac{1}{-j1}+\frac{1}{2} & -1 \\ 1 & -1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} V_{S1}/4 \\ 0 \\ V_{S2} \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ I_S \end{bmatrix}$$

$V_1 = 33.63 + j90.77 \longrightarrow 96.8\angle 69.7^\circ\ V$

$V_2 = -16.37 + j4.17 \longrightarrow 16.89\angle 165.7^\circ\ V$

Now we solve the same problem with mesh analysis



① $-V_{S1} + 4I_1 + j4(I_1 - I_2) = 0$

② $j4(I_2 - I_1) + V_{S2} + -j1\cdot(I_2 - I_3) = 0$

③ $-j1(I_3 - I_2) + I_3 \times 2 = 0$

(1) $[4+j4]I_1 \quad -j4I_2 \quad +0I_3 = V_{s1}$

(2) $-j4I_1 \quad +[j4-j1]I_2 \quad +j1I_3 = -V_{s2}$

(3) $0I_1 \quad +j1I_2 \quad +(2-j)I_3 = 0$

$$\begin{bmatrix} 4+j4 & -j4 & 0 \\ -j4 & j4-j1 & j1 \\ 0 & j1 & 2-j \end{bmatrix}^{-1} \begin{bmatrix} V_{s1} \\ -V_{s2} \\ 0 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$I_1 = 10.34 - j22.69$
$I_2 = -12.35 - j14.28$
$I_3 = -8.18 + j2.08$

$$\underbrace{V_1 = (I_1 - I_2) \times j4}_{\text{see circuit}} = 96.8 \angle 69.7^\circ \text{ V}$$

$$V_2 = [I_2 - I_3] \times -j1 = 16.9 \angle 165.7^\circ \text{ V}$$

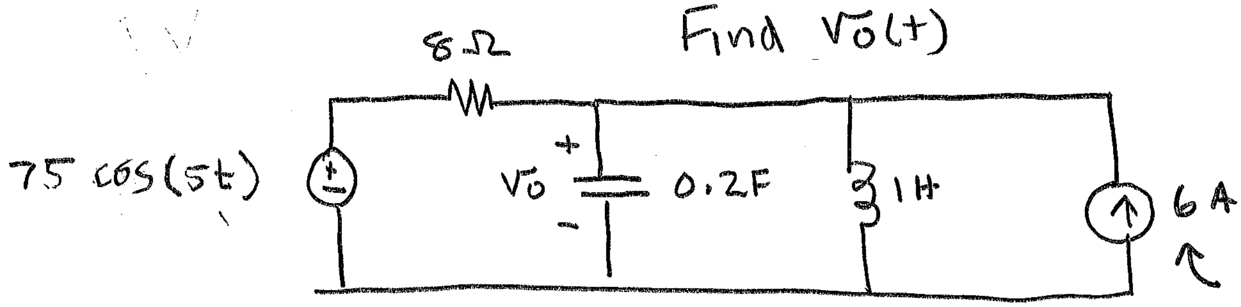
Superposition in AC circuits

For any linear system $f(a+b) = f(a) + f(b)$

This is really useful when circuit has sources with different frequencies.

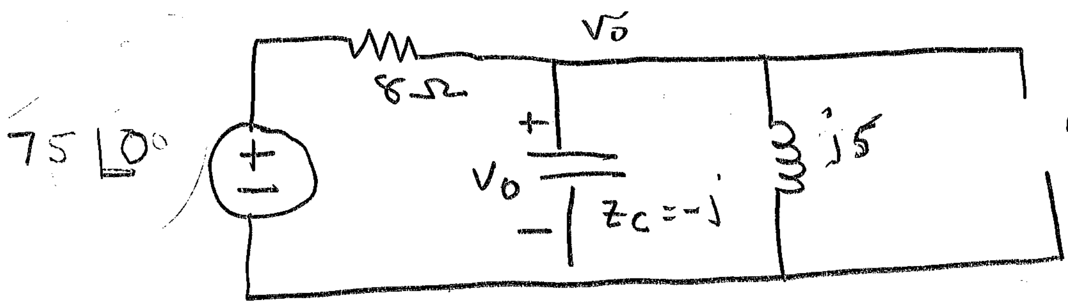
Practice problem 10.6 pg 424 (sort of)

Find $v_o(t)$



← dc is different than example

Get response from AC source



current source disabled.

$$Z_c = \frac{1}{j\omega C} = \frac{1}{j5 \times 0.2} = -j \qquad Z_L = j\omega L = j5$$

$$V_o = \frac{Z_c // Z_L}{Z_c // Z_L + 8} \times 75\angle 0 \qquad \text{Voltage Division}$$

$$Z_c // Z_L = \frac{j5 \times -j}{j5 - j} = \frac{5}{4j} = -1.25j$$

So $V_o = \dfrac{-1.25j}{-1.25j + 8} \times 75\angle 0 = 1.784 - j11.44\ V$

$= 11.58\angle -81.12^\circ\ V$

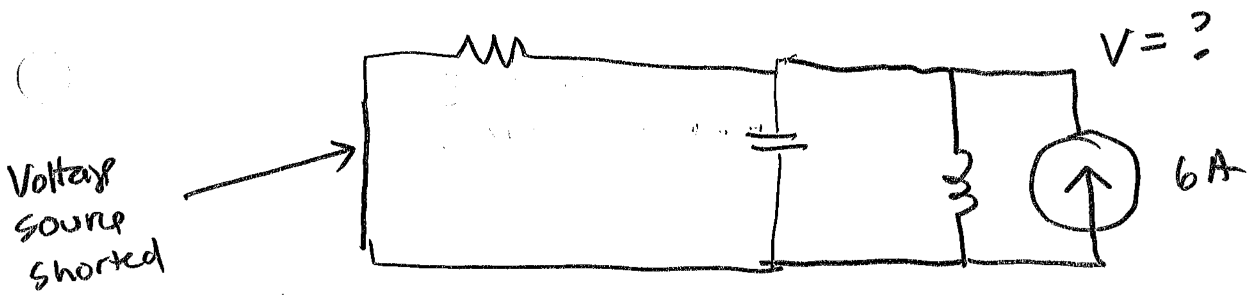
Now use EQ, 9.25

$V(t) = V_m \cos(\omega t + \phi) \Longleftrightarrow V = V_m \angle \phi$

11.58   −81.12

So $V(t) = 11.58 \cos(5t - 81.12^\circ)$

Now solve for output due to DC response

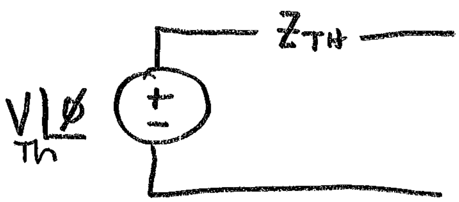


$V(t) = 11.58 \cos(5t - 81.12^\circ)$

Note that I set the source on the right to DC, If we used 6cos(10t) as shown in the book, we would have two sinewaves at different frequencies

Thevenin's theorem

Any linear source can be represented with this circuit:



We can measure V_{Th} by simply observing the open circuit voltage

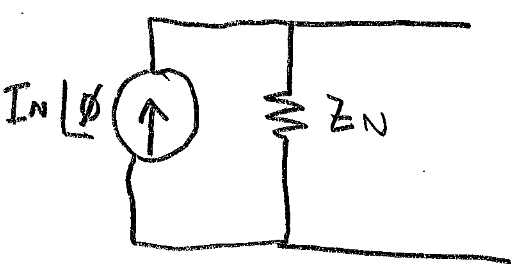
We can measure Z_{TH} by shorting the voltage source and measuring the resistance.

We can also determine Z_{TH} by measuring the short circuit current. Then

$$Z_{Th} = \frac{V_{TH}}{I_{SC}}$$

Norton's Theorem

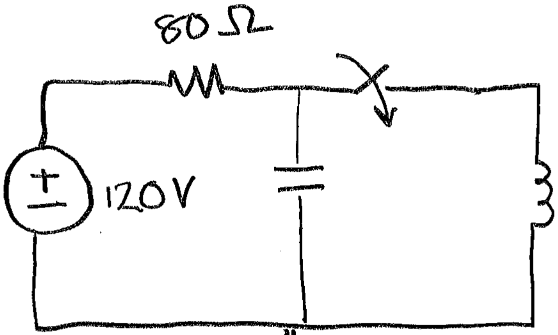
Any linear source can be represented with this circuit:



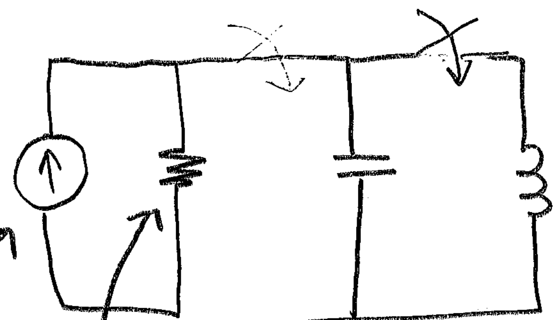
We can determine I_N by shorting the output and measuring the current.

We can determine Z_N by disabling the current source and measuring impedance.

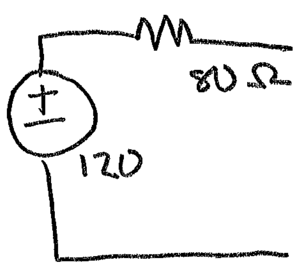
Re-Visit Problem 8.53



"GENERAL"



"PARALLEL RLC"



CONVERT TO NORTON

Short OUTPUT, $\frac{120}{80} = 1.5A$

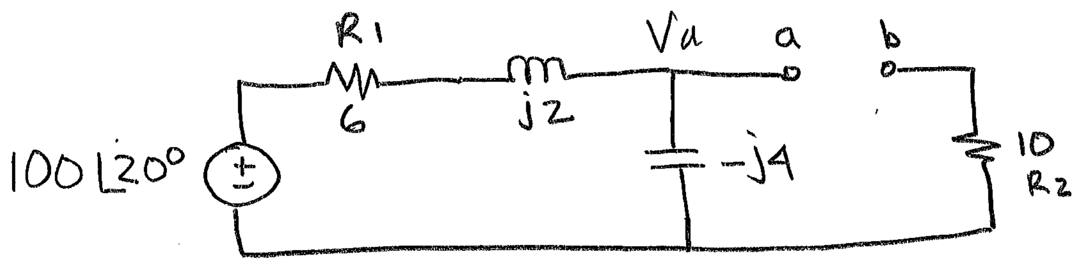
FOR NORTON, we know $I_N \times Z_N = V_{oc}$

so $Z_N = \frac{V_{oc}}{I_N} = \frac{120}{1.5} = 80\Omega$

Simplified the problem.

Practice problem 10.8 pg 428

Find Thevenin equivalent at terminals a-b - also NORTON



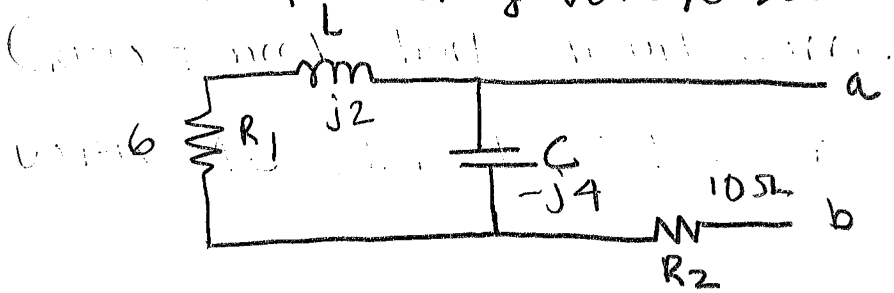
First get open circuit voltage.

No current in 10 Ω resistor

so

$$V_{Th} = \frac{-j4}{6+j2-j4} \times 100\angle 20 = 63.24\angle -51.56^\circ$$

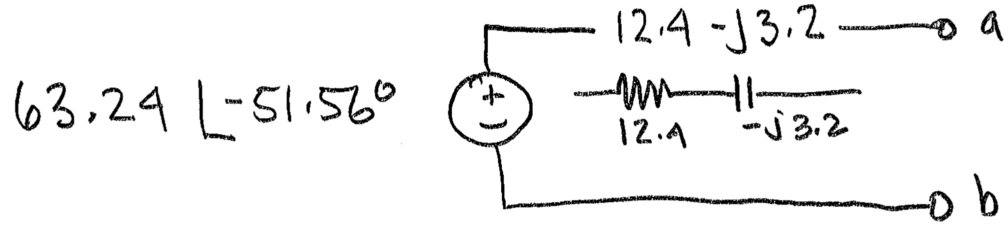
Get Z_{Th} by disabling voltage source and measuring

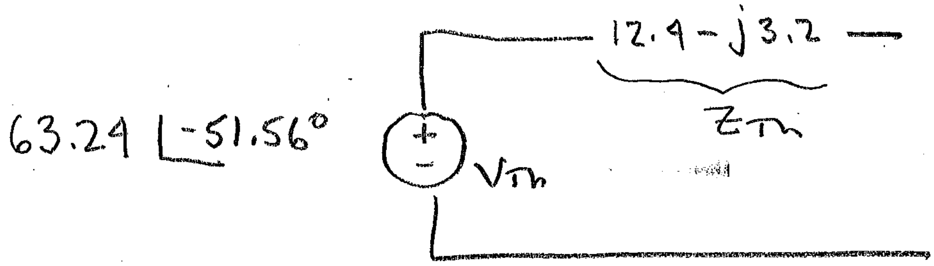


$$Z_{Th} = (R_1 + Z_L) // Z_C + R_2$$

$$= \frac{(6+j2)\times(-j4)}{(6+j2)-j4} + 10 = 12.4 - j3.2\ \Omega$$

Thevenen EQUIV is



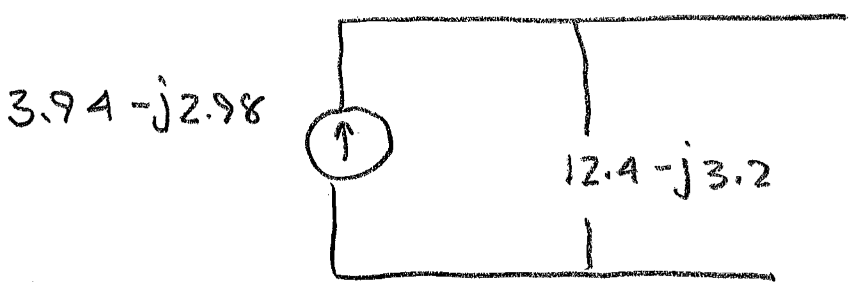


Thevenin Source

$$I_N = \frac{V_{Th}}{Z_{Th}} = \frac{63.24 \angle -51.56}{12.4 - j3.2} = \frac{39.31 - j49.53}{12.4 - j3.2}$$

$$= \underline{3.94 - j2.98 \text{ A}}$$

and $Z_N = Z_{Th} = 12.4 - j3.2$

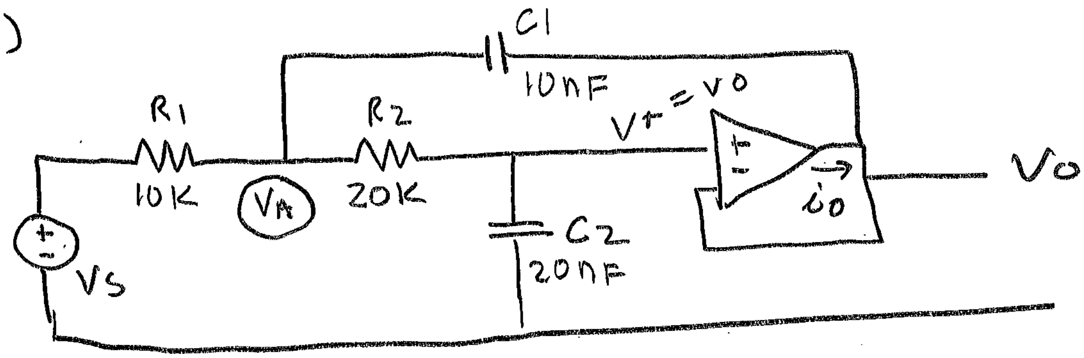


NORTON SOURCE

1D

PRACTICE PROBLEM 10.11 Pg 432

Find $V_o(t)$ $i_o(t)$



KCL @ node A

$V^+ = V^-$

(1) $$\frac{V_S - V_A}{R_1} + \frac{V_O - V_A}{Z_{C1}} + \frac{V_O - V_A}{R_2} = 0$$

UNK V_O, V_A

KCL at non-inv terminal

(2) $$\frac{V_O - V_A}{R_2} + \frac{V_O}{Z_{C2}} = 0$$

(1) ↓

$$V_A\left[-\frac{1}{R_1} - \frac{1}{Z_{C1}} - \frac{1}{R_2}\right] + V_O\left[\frac{1}{Z_{C1}} + \frac{1}{R_2}\right] = -\frac{V_S}{R_1}$$

(2) $$V_A\left[-\frac{1}{R_2}\right] + V_O\left[\frac{1}{R_2} + \frac{1}{Z_{C2}}\right] = 0$$

so

$$\begin{bmatrix} -\frac{1}{R_1} - \frac{1}{Z_{C1}} - \frac{1}{R_2} & \frac{1}{Z_{C1}} + \frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{Z_{C2}} \end{bmatrix}^{-1} \begin{bmatrix} -\frac{V_S}{R_1} \\ 0 \end{bmatrix} = \begin{bmatrix} V_A \\ V_O \end{bmatrix}$$

$R_1 = 10K, \quad R_2 = 20K, \quad C_1 = 10nF, \quad C_2 = 20nF$

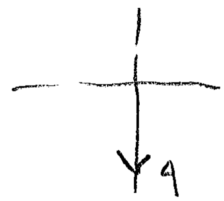
$V_s = 12\angle 0, \quad Z_{C_1} = \dfrac{1}{j\omega C_1} = \dfrac{1}{j \times 5000 \times 10nF} = -j20K$

$Z_{C_2} = \dfrac{1}{j\omega C_2} = \dfrac{1}{j \cdot 5000 \times 20nF} = -j10K$

substituting in the matrix:

$V_A = 8 - j4$

$V_o = -j4 = 4\angle -90$



Use 9.25 (Pg 379)

$V_o = 4\angle -90 \Longleftrightarrow 4\cos(5000t - 90)$

$V_o = 4\sin(5000t)$

Now get $i_o(t)$

No current into op-amp terminal V^-

So $I_o = \dfrac{V_o - V_A}{Z_{C_1}}$ ← see circuit

$= \dfrac{-j4 - (8 - j4)}{-j20K} = \dfrac{-8}{-j20K} = -j\,400E{-6}$

$= 400E{-6}\angle -90$

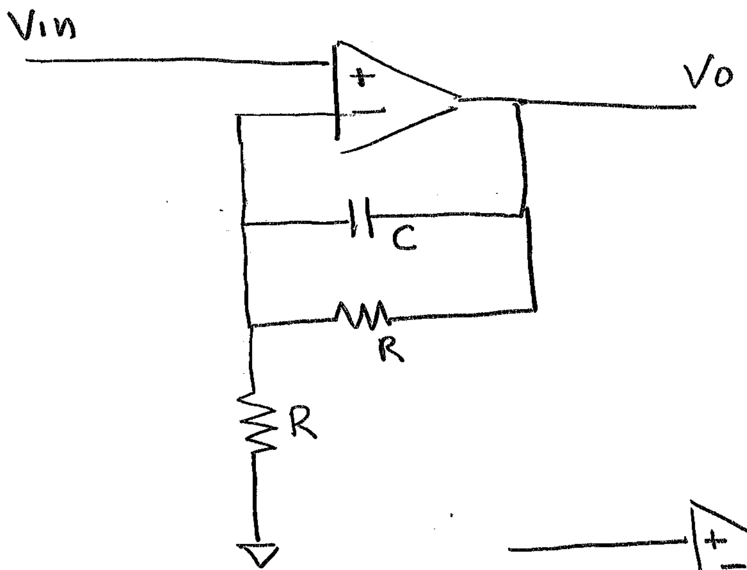
$= 400E{-6}\sin(5000t)$

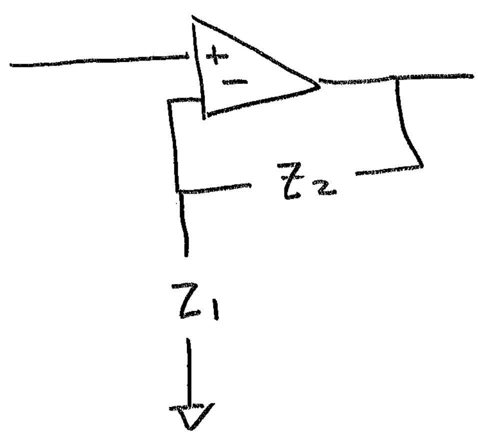
EASY OP-AMP PROBLEM

Practice Problem 10.12

find Gain and phase shift

? ?





Non-inverting Configuration

$$\frac{V_o}{V_{in}} = 1 + \frac{Z_2}{Z_1}$$

$$Z_2 = Z_R \,//\, Z_C$$

$$Z_2 = \frac{R \times \frac{1}{j\omega C}}{R + 1/j\omega C} =$$

$$Z_2 = \frac{R}{j\omega RC + 1}$$

So $\frac{V_o}{V_{in}} = 1 + \frac{Z_2}{Z_1} = 1 + \frac{\frac{R}{j\omega RC + 1}}{R}$

$$= 1 + \frac{1}{j\omega RC + 1}$$

With $R = 10K$, $C = 1uf$ and $\omega = 1000\,r/s$

$$\frac{V_o}{V_{in}} = 1 + \frac{1}{j\omega RC + 1} = 1 + \frac{1}{j \times 1000 \times 10K \times 1uF + 1}$$

$$= 1.0099 - j0.0990$$

$$= 1.0147 \angle -5.6^\circ$$

"Gain" "phase"

will |Gain| increase or decrease if frequency increases?

what is the Gain at DC?

what is the gain at $\omega = \infty$?