LECTURE #6

GENERALIZED SECOND ORDER CIRCUITS

SEC 8.7

We have worked with series RLC and parallel RLC.

Now we will work with any circuit with two energy storage elements.

STEPS (Pg 339)

1) find $r(0^+)$ $\frac{dr}{dt}(0^+)$, and $r(\infty)$

(These can be interchanged)

2) Turn off independent sources, use KVL/KCL to write a differental equation. Obtain the transient response

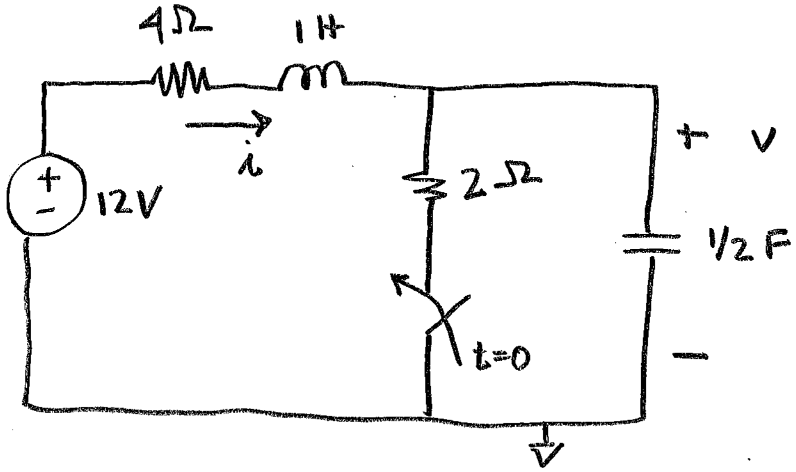
3) Add in the steady state response, $r_{ss} = r(\infty)$

4) Final result $r(t) = r_t(t) + r_{ss}$

transient steady state

Example 8.9 Pg 339

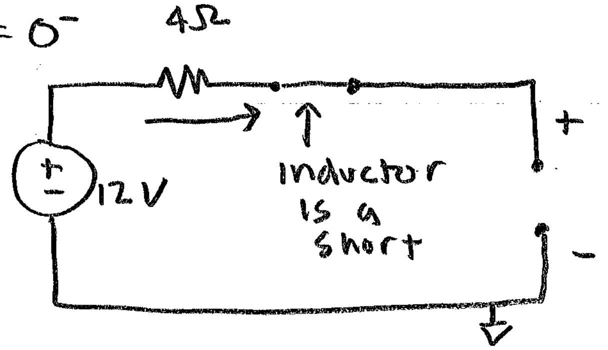
(Fi)nd the complete response for v and then i



Neither series nor parallel RLC

Step 1 - Initial and final conditions

At $t = 0^-$



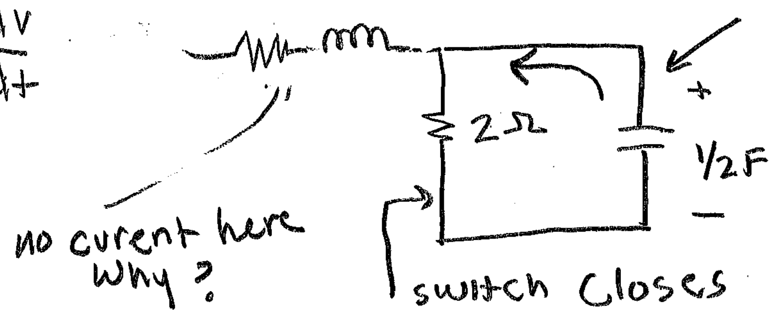
Capacitor is open

so $V(0^+) = V(0^-) = 12\,V$

$i(0^+) = i(0^-) = 0\,A$

Get $\frac{dv}{dt}$

$V(0^+) = V(0^-) = 12V$



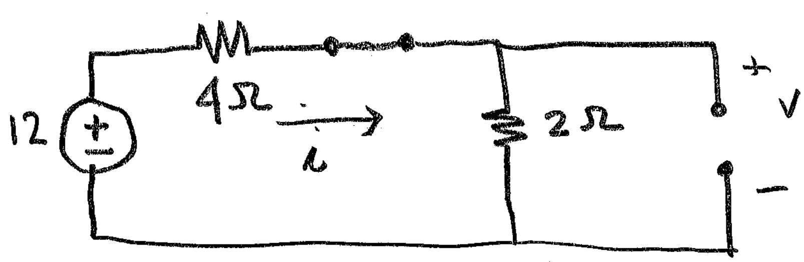
Capacitor current $= -\frac{12}{2} = -6A$, $\frac{dv}{dt} = \frac{i}{C} = \frac{-6}{1/2} = -12\,\frac{V}{S}$

Get $\frac{di}{dt}$ $\qquad \frac{di_L(0^+)}{dt} = \frac{V_L(0^+)}{L}$

Capacitor voltage does not change, 12 V source doesn't change

So inductor voltage doesn't change and $\frac{di}{dt} = 0$ A/s

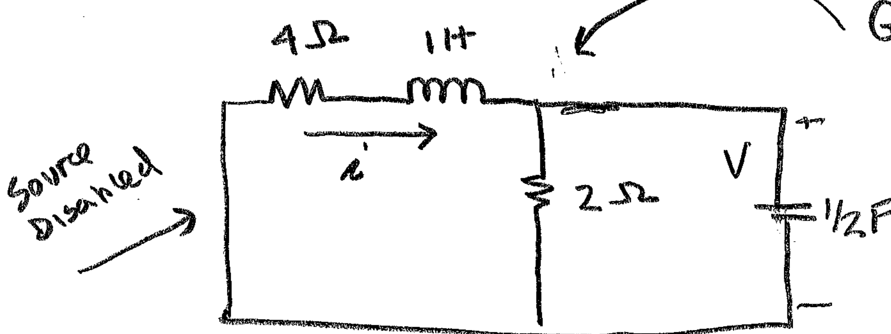
Final conditions



$V(\infty) = \frac{2}{4+2} \times 12 = 4.V$

$i(\infty) = \frac{12}{4+2} = 2A$

STEP 2



Get transient with KCL at this node

KCL

$-i + \frac{V}{2} + C\frac{dv}{dt} = 0$ ← sum currents leaving node to zero

This is one equation in two unknowns.

write KVL in left mesh (i is mesh current)

$4i + L\frac{di}{dt} + V = 0$ so now we have 2 eq in 2 unk.

EQUATIONS

① $i = \frac{V}{2} + C\frac{dv}{dt}$ ← from KCL

② $4i + L\frac{di}{dt} + V = 0$ ← from KVL

Let's substitute ① into ②

$$\underbrace{4\left[\frac{V}{2} + C\frac{dv}{dt}\right]}_{i} + L\underbrace{\left[\frac{1}{2}\frac{dv}{dt} + C\frac{d^2v}{dt}\right]}_{\frac{d}{dt}\text{ of ① is } di/dt} + V = 0$$

← UGLY, but 1 EQ in 1 UNK

$$-C\frac{d^2v}{dt} + \left(4C + \tfrac{1}{2}L\right)\frac{dv}{dt} + 3V = 0$$

$$\frac{1}{2}\frac{d^2v}{dt} + 2.5\frac{dv}{dt} + 3V = 0$$

see eq 8.7 for this step

$$\frac{d^2v}{dt} + 5\frac{dv}{dt} + 6 = 0 \implies s^2 + 5s + 6 = 0$$

roots are $-2, -3$

so $V_n(t) = Ae^{-2t} + Be^{-3t}$

natural response eq 8.9.6

Both roots are distinct and real so overdamped (Book doesn't say this)

Important note: I could have picked ANY voltage or current and I would get the same characteristic Eqn and same s_1, s_2. (Book doesn't emphasise this)

Use the equation from page 2 of lecture #3 for overdamped circuit.

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ s_1 & s_2 \end{bmatrix}^{-1} \begin{bmatrix} r(0^+) - R_s \\ dr/dt\,(0^+) \end{bmatrix}$$

solve for $V(t)$

transient

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}^{-1} \times \begin{bmatrix} 8 \\ -12 \end{bmatrix} \begin{matrix} \leftarrow V(0^+) - V_\infty \\ \leftarrow \frac{dv}{dt}(0^+) \end{matrix}$$

So $A_1 = 12$, $A_2 = -4$

$V(t) = 12e^{-2t} - 4e^{-3t}$ ← transient voltage response

Now get transient current response

$i_n(t) = A_1 e^{-2t} + A_2 e^{-3t}$

Same procedure as above

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}^{-1} \begin{bmatrix} -2 \\ 0 \end{bmatrix} \begin{matrix} - i(0^+) - i_\infty \\ - \frac{di}{dt}(0^+) \end{matrix}$$

$A_1 = -6$, $A_2 = 4$

$i_n(t) = -6e^{-2t} + 4e^{-3t}$ ← transient current response

Step 3 add in steady state

$$v(t) = \underset{V_\infty}{4} + 12e^{-2t} - 4e^{-3t}$$

$$i(t) = \underset{i_\infty}{2} - 6e^{-2t} + 4e^{-3t}$$

Quick check:

we know $V(0) = 12\,V$

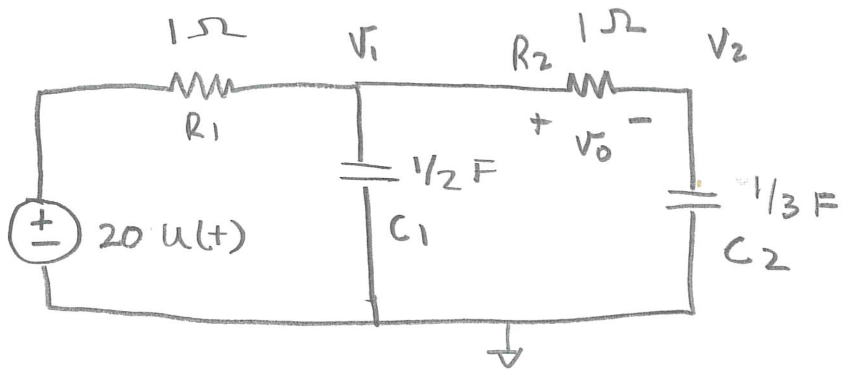
$$V(0) = 4 + 12 \times e^0 - 4e^0 = 12 \checkmark$$

we know $i(0) = 0\,A$

$$i(0) = 2 - 6e^0 + 4e^0 = 0 \checkmark$$

We could check with $\frac{d}{dt}$ of the equations also

PRACTICE PROBLEM 8.10, PG 343 | 6½



Find $V_0(t)$ Hint: (First find V_1 and V_2)

① $\frac{V_1}{1} + C_1\frac{dV_1}{dt} + \frac{(V_1 - V_2)}{1} = 0$ } Where does this equation come from?

disable 20V

in cap and resistor

r V_2, I know $i = C\, dV/dt$

So

② $\underbrace{C_2\frac{dV_2}{dt} \times R}_{\text{Voltage in resistor}} + V_2 = V_1$ } where does this equation come from?

V_2 is voltage across C_2

Substitute ② into ①

① $\underbrace{\left[C_2\frac{dV_2}{dt}R + V_2\right]}_{\text{Subst for } V_1} + C_1\underbrace{\left[C_2\frac{d^2V_2}{dt}\times R + \frac{dV_2}{dt}\right]}_{dV_1/dt} + \left[C_2\frac{dV_2}{dt}R + V_2\right] - V_2 = 0$

UGLY BUT
1 EQ IN 1 UNK

$$C_1 C_2 \frac{d^2 V_2}{dt} + (C_2 + C_1 + C_2) \frac{dV_2}{dt} + V_2 = 0$$

$$\frac{1}{6} \frac{d^2 V_2}{dt} + \left(\frac{2}{3} + \frac{1}{2}\right) \frac{dV_2}{dt} + V_2 = 0$$

$$\frac{d^2 V_2}{dt} + 7 \frac{dV_2}{dt} + 6V_2 = 0$$

$s^2 + 7s + 6 = 0 \Longleftarrow$ Characteristic EQN

roots are $-1, -6$

At this point we know the roots. Furthermore they are real and distinct, so circuit response is over damped

$$V_0 = V_{0ss} + A_1 e^{-t} + A_2 e^{-6t}$$

Get initial and final conditions

$V_0(0^-) = 0$, No current in cap

$V_0(\infty) = 0$, no current in cap

$\frac{dV_0(0^+)}{dt} = \frac{40V}{s}$ Why? $V_{C_1}(0^+) = V_{C_1}(0^-) = 0$

$i_{C_1} = \frac{20V}{1\,\Omega} = 20\text{ A}$; $\frac{dV_{C_1}}{dt} = \frac{20}{1/2} = 40V/s$

$i_{C_2}(0^+) = 0$, $\frac{dV_{C2}}{dt} = 0$ so $\frac{dV_0}{dt} = 40\,V/s$

Use our equation for overdamped.

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ s_1 & s_2 \end{bmatrix}^{-1} \begin{bmatrix} v(0^+) - V_\infty \\ dv/dt(0^+) \end{bmatrix}$$ which is zero

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -6 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 40 \end{bmatrix} = \begin{bmatrix} 8 \\ -8 \end{bmatrix}$$

$A_1 = 8$, $A_2 = -8$

and so

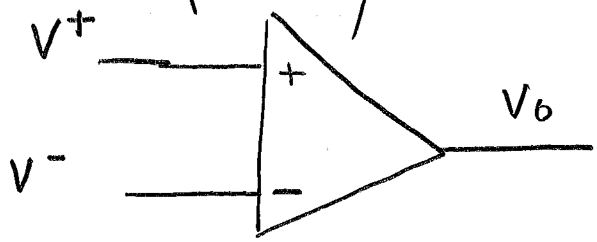
$$V_0 = 8e^{-t} - 8e^{-6t}$$

INTRODUCTION to OP-AMPS FOR EE-310

No current flows into inputs V^- and V^+

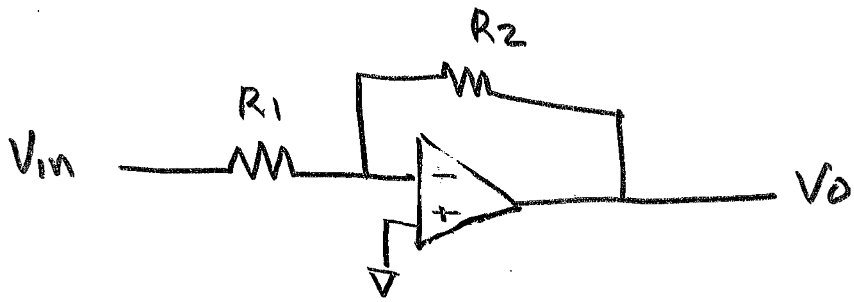
$V_o = \text{Gain} \cdot (V^+ - V^-)$

Gain of op-amp is infinite



IF OUTPUT is between -15 and $+15\,V$, $V^+ = V^-$

Gain <u>of op amp</u> is infinite. What is the gain of this circuit?



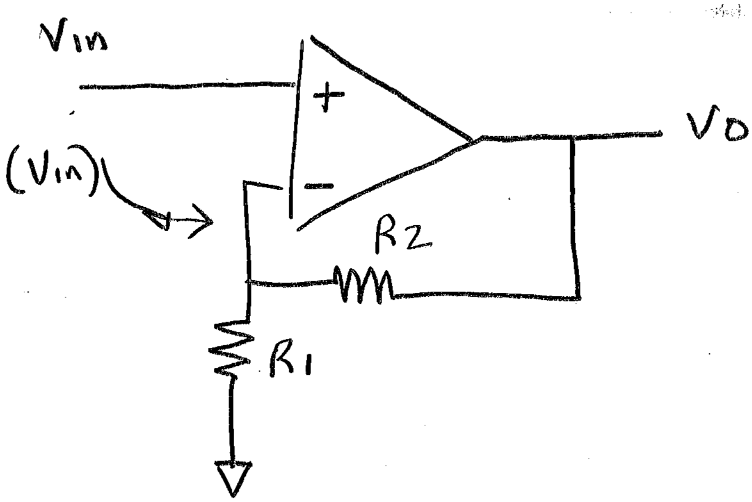
KCL at inverting terminal: (No current into V^-)

Also, since $V^+ = V^-$, $V^- = 0$

$$\frac{V_{in}}{R_1} + \frac{V_o}{R_2} = 0, \quad \frac{V_{in}}{R_1} = -\frac{V_o}{R_2} \quad \text{or Gain} = \frac{V_o}{V_{in}} = -\frac{R_2}{R_1}$$

"Inverting Configuration"

What is the gain of this circuit ?



Use KCL again. Remember $V^+ = V^- = V_{in}$

$$\frac{V_{in}}{R_1} = \frac{V_o - V_{in}}{R_2}$$

$$V_{in} R_2 = R_1 (V_o - V_{in}) = R_1 V_o - R_1 V_{in}$$

$$V_o R_1 = V_{in} R_2 + V_{in} R_1$$

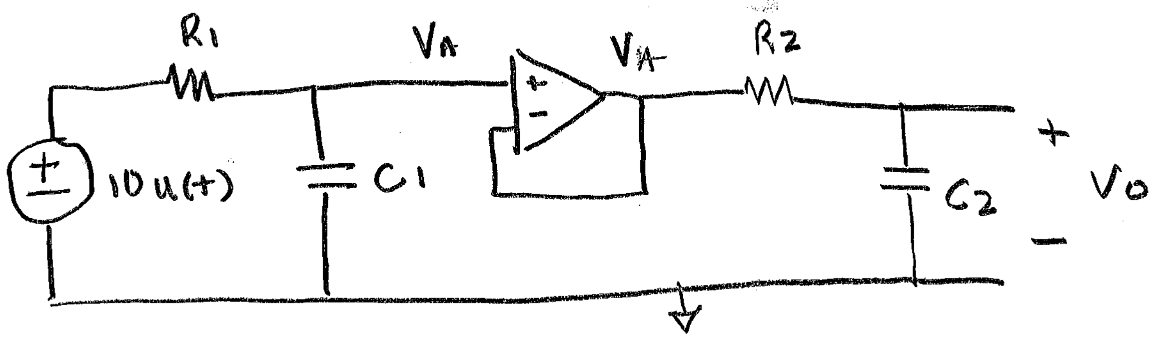
$$V_o R_1 = V_{in} (R_2 + R_1)$$

or $$\frac{V_o}{V_{in}} = \text{Gain} = \frac{R_2 + R_1}{R_1}$$

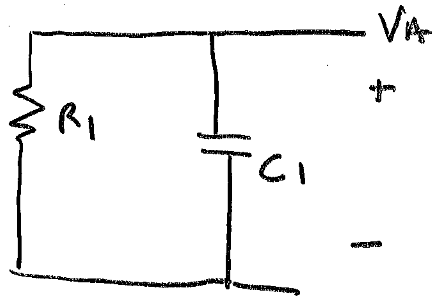
"Non-inverting configuration"

PRACTICE PROBLEM 8.11, PG 346

find $V_o(t)$



STEP 1 − GET CHARACTERISTIC EQN AND SOLVE.



$$① \quad \frac{V_A}{R_1} + \underbrace{C_1 \frac{dV_A}{dt}}_{\text{capacitor current}} = 0$$

Since OP AMP GAIN IS 1,

$$② \quad \underbrace{\frac{V_A - V_o}{R_2}}_{\text{current in } R_2} = \underbrace{C_2 \frac{dV_o}{dt}}_{\text{Current in } C_2}$$

$$\underbrace{① \quad \frac{V_A}{R_1} + C_1 \frac{dV_A}{dt} = 0 \qquad ② \quad \frac{V_A - V_o}{R_2} = C_2 \frac{dV_o}{dt}}_{\text{2 EQ IN 2 UNKNOWNS}}$$

I can solve ② for V_A

$$② \quad V_A = R_2 C_2 \frac{dV_o}{dt} + V_o$$

Substitute (2) into 1

$$\underbrace{\frac{1}{R_1}\left[R_2C_2\frac{dv_o}{dt}+v_o\right]}_{V_A} + \underbrace{C_1\left[R_2C_2\frac{d^2v_o}{dt}+\frac{dv_o}{dt}\right]}_{dV_A/dt} = 0$$

Yecch!!!

$$R_2C_1C_2\frac{d^2v_o(t)}{dt} + \left(\frac{R_2C_2}{R_1}+C_1\right)\frac{dv_o(t)}{dt} + \frac{v_o(t)}{R_1} = 0$$

$R_1 = R_2 = 10K$, $C_1 = 20\,uF$, $C_2 = 100\,uF$

$$20E{-6}\,\frac{d^2V_o(t)}{dt} + 120E{-6}\,\frac{dV_o(t)}{dt} + 100E{-6}\,V_o(t)$$

$$\frac{d^2V_o(t)}{dt} + 6\frac{dV(t)}{dt} + 5V(t) = 0$$

So $\quad s^2+6s+5=0$, $\quad s_{1,2} = -1, -5$

STEP 2 Get initial and final conditions

$V_o(0^+) = 0 \quad$ Why?

$\frac{d(V_o)}{dt}(0^+) = 0 \quad$ Voltage at C_1 is 0 at $t = 0^+$.
$\therefore$ no current into C_2

$V_o(\infty) = 10\,V \quad$ Why?

roots are distinct and real, so over damped

STEP 3 Get A_1, A_2

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -5 \end{bmatrix}^{-1} \begin{bmatrix} -10 \\ 0 \end{bmatrix}$$

$\underset{V_o(0^+)}{0} - \underset{V_\infty}{10}$

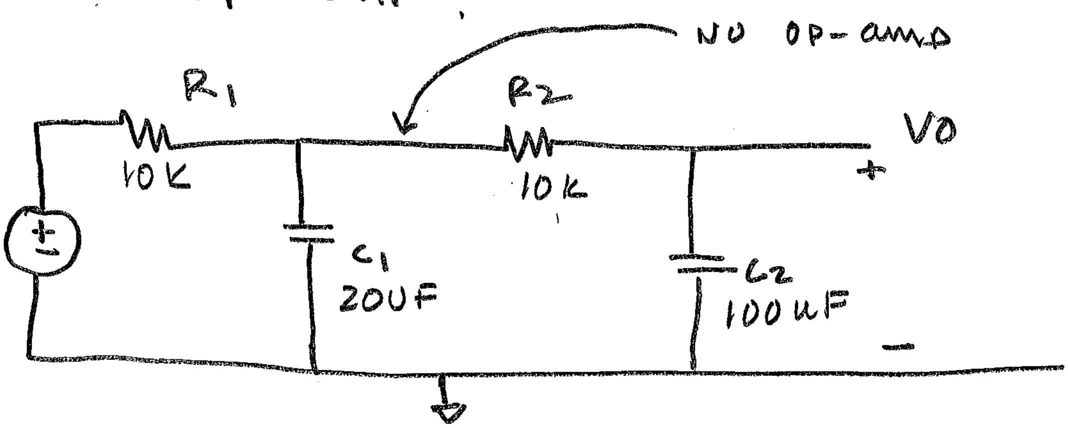
$-\frac{dV_o}{dt}(0^+)$

$A_1 = -12.5$

$A_2 = 2.5$

$$V_o(t) = 10 - 12.5e^{-t} + 2.5e^{-5t}$$

what is the difference between the circuit we analyzed and the same circuit, but without the op-amp?



Generalized responses

Over damped

$$r(t) = R_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$r(0^+) = R_s + A_1 + A_2$$

$$dr/dt\,(0^+) = A_1 s_1 + A_2 s_2$$

so
$$\begin{bmatrix} 1 & 1 \\ s_1 & s_2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} r(0^+) - R_s \\ dr/dt(0^+) \end{bmatrix}$$

or
$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ s_1 & s_2 \end{bmatrix}^{-1} \begin{bmatrix} r(0^+) - R_s \\ dr/dt(0^+) \end{bmatrix}$$

Critically damped

$$r(t) = R_s + (A_1 + A_2 t)e^{-\alpha t}$$

$$\left.\begin{aligned} r(0^+) &= R_s + A_1 \\ \frac{dr}{dt}(0^+) &= -\alpha A_1 + A_2 \end{aligned}\right\}$$

so $A_1 = r(0^+) - R_s$

$$A_2 = \frac{dr}{dt}(0^+) + \alpha A_1$$

Underdamped

$$r(t) = R_s + (A_1 \cos(w_d t) + A_2 \sin(w_d t))e^{-\alpha t}$$

$$r(0^+) = R_s + A_1$$

$$\frac{dr}{dt} = \frac{d}{dt}\left[R_s + A_1 \cos(w_d t)e^{-\alpha t} + A_2 \sin(w_d t)e^{-\alpha t}\right]$$

$$= -\alpha A_1 \cos(w_d t)e^{-\alpha t} - A_1 w_d \sin(w_d t)e^{-\alpha t}$$

$$+ A_2 w_d \cos(w_d t)e^{-\alpha t} - \alpha A_2 \sin(w_d t)e^{-\alpha t}$$

$$\frac{dr}{dt}(0^+) = -\alpha A_1 + A_2 w_d$$

so $A_1 = r(0^+) - R_s$

$$A_2 = \frac{\frac{dr(0^+)}{dt} + \alpha A_1}{w_d}$$