LECTURE #5

STEP RESPONSE OF SERIES AND PARALLEL RLC CIRCUITS Sec 8.5 & 8.6

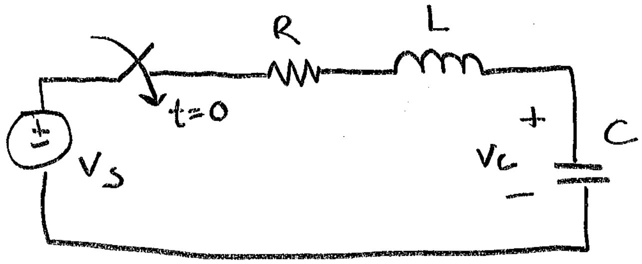


FIG 8.39

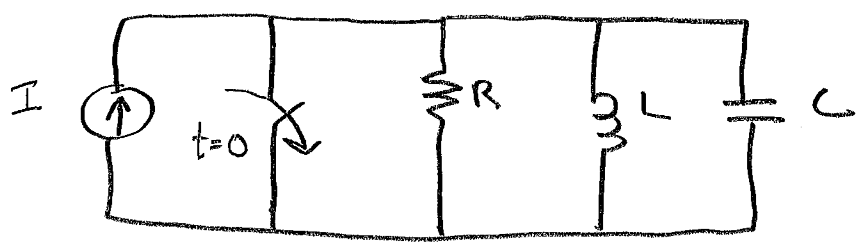
What's different?

What's the final capacitor voltage?

In this case we are applying energy to the circuit at $t=0$

What is the energy in the inductor at $t=\infty$?

What is the energy in the capacitor at $t=\infty$?



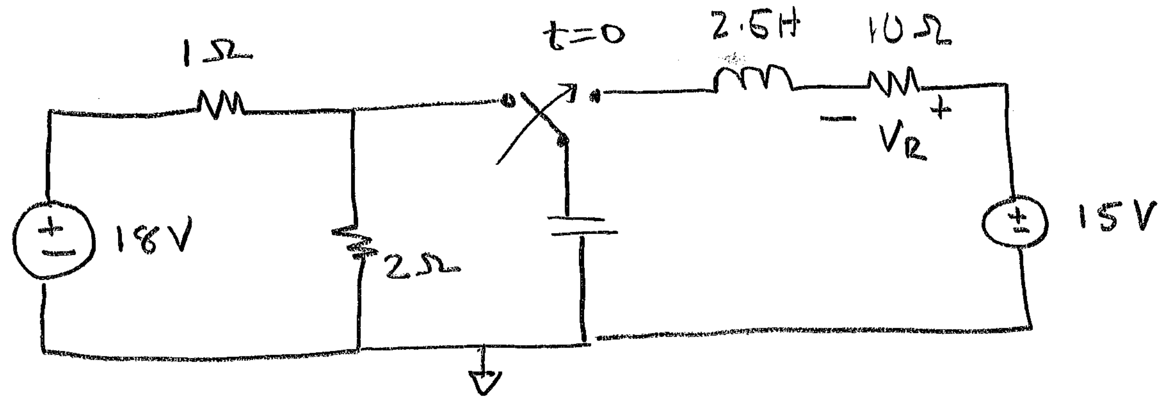
What is the energy in the inductor at $t=\infty$?

What is the energy in the capacitor at $t=\infty$?

We will update our equations to include a non-zero response at $t=\infty$

See page [illegible]

Does the additional source change s_1, s_2 ?



At $t > 0$, V_R is affected by two sources

1) The transient response from the initial Voltage in the Capacitor

2) The +15 Volt source

Two sources? use superposition

$$V_R(t) = V_t(t) + V_{ss} \leftarrow \text{EQ } 8.41$$

$V_t(t)$: Voltage due to transient, dies out with time.

V_{ss}: steady state voltage due to 15 V

for transient, the 15V supply is disabled, (set to zero).

- The characteristic equation is unchanged.
- The natural frequencies (s_1, s_2) are unchanged

$r(t) = R_s + A_1 e^{st} + A_2 e^{st}$ (overdamped)

$r(t) = R_s + e^{-\alpha t}(A_2 + A_1 t)$ (critically damped)

$r(t) = R_s + (A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t))\, e^{-\alpha t}$ (underdamped)

if $r(t)$ is a voltage, this is Eq 8.44
if $r(t)$ is a current this is EQ 8.49

$r(t)$ can be any voltage or current in the circuit.

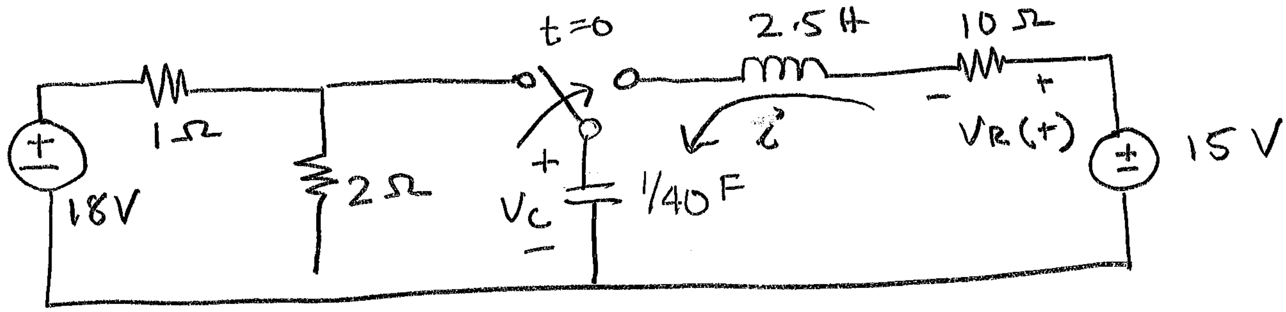
R_s is the steady-state ($t=\infty$) value of the variable.

set the initial conditions ($r(0^+)$, $\frac{dr}{dt}(0^+)$) and the final condition $r(\infty)$, and these equations will work!

So, to compute R_s, A_1, A_2, we need

$r(0^+)$, $\frac{dr}{dt}(0^+)$, and $r(\infty)$

Practice problem 8.7 Pg 336



Find $V_c(t)$ and $V_R(t)$

1) Triage the problem

- It's a series RLC
- We are asked for $V_R(t)$ and $V_c(t)$

Our equations for $R(t)$ are good for any voltage or current in the circuit. I could just work the problem twice.

But if I solve for V_c first, I can get

$$i_c(t) = i_R(t) = C\frac{dV_c(t)}{dt}$$

and so $V_R = i \times R = RC\frac{dV_c(t)}{dt}$

It's much easier to compute the derivative than to do the problem twice.

2) Determine the response type and s_1, s_2

$$\underbrace{\alpha = \frac{R}{2L} = \frac{10}{2 \times 2.5} = 2 \qquad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.5 \times 1/40}} = 4}_{\text{EQ 8.11 for series RLC}}$$

$\alpha < \omega_0$ so underdamped. → Boing

and $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{4^2 - 2^2} = \sqrt{12}$

$= 3.46$ rad/sec

3) Get initial conditions $v(0^+)$ $dv/dt(0^+)$
and final condition $V_C(\infty)$

$$V_C(0^+) = V_C(0^-) = \frac{2}{2+1} \times 18 = 12\,V$$

since $i_C = C\frac{dv}{dt}$, $\frac{dv}{dt}(0^+) = \frac{i_C(0^+)}{C}$

$i_C(0^-) = 0$

$i_L(0^+) = i_L(0^-)$ so $i_C(0^+) = 0$

and $\frac{dV_C}{dt}(0^+) = \frac{i_C(0^+)}{C} = \frac{0}{C} = 0$

So $V_C(0^+) = 12\,V$, $\frac{dV_C}{dt}(0^+) = 0$

$V_C(\infty) = 15\,V$ why?

4) Compute the constants A and B

Use EQ 8.49 or the equations on sheet 2

$$v(t) = V_s + \left(A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)\right) e^{-\alpha t}$$

$$v(0) = V_s + A_1$$

$$\frac{dv}{dt}(0) = -\alpha A_1 + A_2 \omega_d \longleftarrow$$ See sheet 10 of Lecture #3

So $A_1 = v(0) - V_s = 12 - 15 = -3$

$$A_2 = \frac{\frac{dv}{dt}(0^+) + \alpha A_1}{\omega_d} = \frac{0 + (2 \times -3)}{3.46} = -1.732$$

and we have

$$V_c(t) = 15 + \left(-3 \cos(3.46t) - 1.73 \sin(3.46t)\right) e^{-2t}$$

Now we will solve for $V_R(t)$
from our initial conditions

$$V_R = iR = C \frac{dv(t)}{dt} \times R$$

$\longrightarrow$

The derivative $\frac{dv(t)}{dt}$ will be messy because of the Product rule. I prefer letters to numbers

$$V_C(t) = V_s + (A_1\cos(\omega_d t) + A_2\sin(\omega_d t))e^{-\alpha t}$$

$$\frac{dv_C}{dt} = -\alpha(A_1\cos(\omega_d t) + A_2\sin(\omega_d t))e^{-\alpha t} + e^{-\alpha t}(-A_1\omega_d\sin(\omega_d t) + A_2\omega_d\cos(\omega_d t))$$

$$= -\alpha A_1 e^{-\alpha t}\cos(\omega_d t) - \alpha A_2 e^{-\alpha t}\sin(\omega_d t) - A_1 e^{-\alpha t}\omega_d\sin(\omega_d t) + A_2 e^{-\alpha t}\omega_d\cos(\omega_d t)$$

$$\frac{v_C}{dt} = e^{-\alpha t}\left[\underbrace{\cancel{(-\alpha A_1 + A_2\omega_d)}}_{0\ (=dv/dt)\text{ by substitution}}\cos(\omega_d t) + \underbrace{(-\alpha A_2 - A_1\omega_d)}_{13.86}\cdot\sin(\omega_d t)\right]$$

and

$$V_R(t) = RC\frac{dv_C}{dt} = \underbrace{10 \times \frac{1}{40} \times 13.86}_{3.46} e^{-2t}\sin(3.46t)$$

$$\boxed{V_R = 3.46\, e^{-2t}\sin(3.46t)}$$

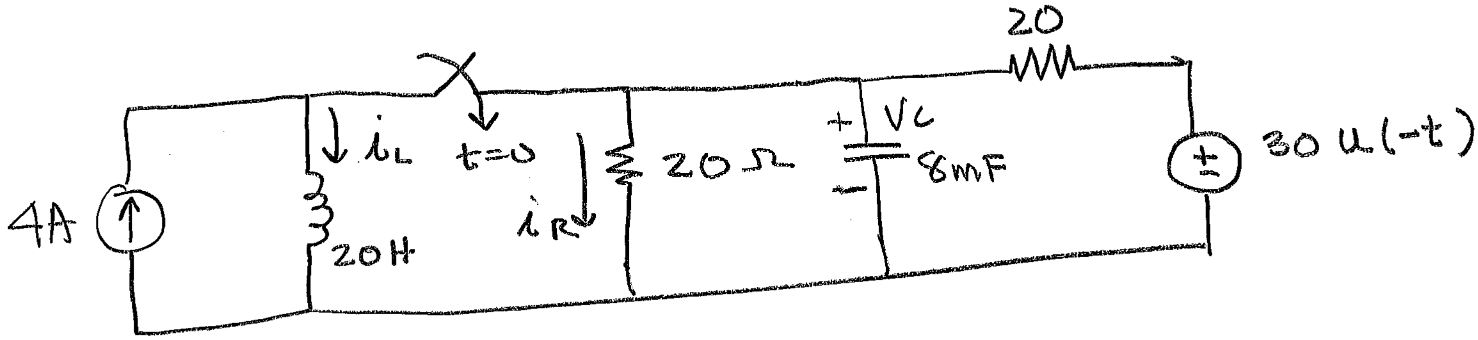
Reasonable? $V_R(0^+) =$ Compare to circuit

$V_R(\infty) =$ Compare to circuit

Was our strategy good? Should we have repeated problems rather than taking dv/dt?

Step Response of Parallel RLC

Example 8.8 pg 337



find $i_L(t)$ and $i_R(t)$

1) Triage

- It's a parallel RLC
- We are asked for $i_L(t)$ and $i_R(t)$
- R is $20 \| 20 = 10\Omega$ for transient response

We could work the problem twice or...

solve for $i_L(t)$

$$V_R(t) = V_L(t) = L\frac{di_L}{dt}$$

$$\text{and } i_R(t) = \frac{V_R(t)}{R} = \frac{L}{R}\frac{di_L}{dt}$$

so would have to take another derivative

... that was nasty - lets just do problem twice!

2) Determine response type and s_1, s_2

$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 10 \times 0.008} = 6.25$ EQ 8.32 for parallel RLC

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 0.008}} = 2.5$ EQ 8.32

Since $\alpha > \omega_0$ it is overdamped — "Thvd"

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -6.25 \pm 5.728$

$= -11.98, \; -0.5218$

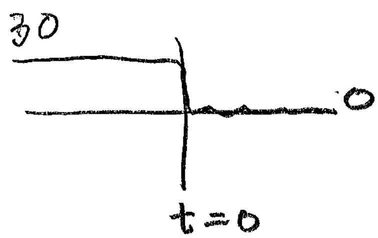
3) Compute initial and final conditions for $i_L(t)$

$i_L(0^+) = i_L(0^-) = \underline{4\,A}$

$i_L(\infty) = \underline{4\,A}$

3 things we need!

voltage source $u(-t)$ is



So at $t = 0^-$, $V_C = \frac{20}{20+20} \times 30 = 15\,V$

when switch closes, $V_C(0^+) = V_C(0^-) = 15\,V$

and $V_L = 15\,V$

So $\frac{di_L(0^+)}{dt} = \frac{V_L(0^+)}{L} = \frac{15\,V}{20\,H} = \underline{0.75\,A/s}$

4) Determine constants

since overdamped

$$i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$i(t) = I_s + A_1 e^{-11.98t} + A_2 e^{-0.5218t}$$

substitute $t = \infty$

$$i_L(\infty) = I_s \quad \text{so} \quad I_s = 4\text{ A}$$

Substitute $t = 0^+$

$$i(0^+) = I_s + A_1 + A_2 \Rightarrow 4 = 4 + A_1 + A_2$$

compute $\frac{di}{dt}(0^+) = -11.98 A_1 e^{-11.98 \times 0} - 0.5218 A_2 e^{-0.5218 \times 0}$

$$-11.98 A_1 - 0.5218 A_2 = \underbrace{0.75}_{\frac{di}{dt}(0^+)}$$

$$A_1 + A_2 = 0$$

so $\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} -11.98 & -0.5218 \\ 1 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 0.75 \\ 0 \end{bmatrix}$

$$A_1 = -0.0655, \quad A_2 = 0.0655$$

so $\boxed{i_L(t) = 4 + .0655\left(e^{-0.5218t} - e^{-11.98t}\right)}$

Now we compute $i_R(t)$

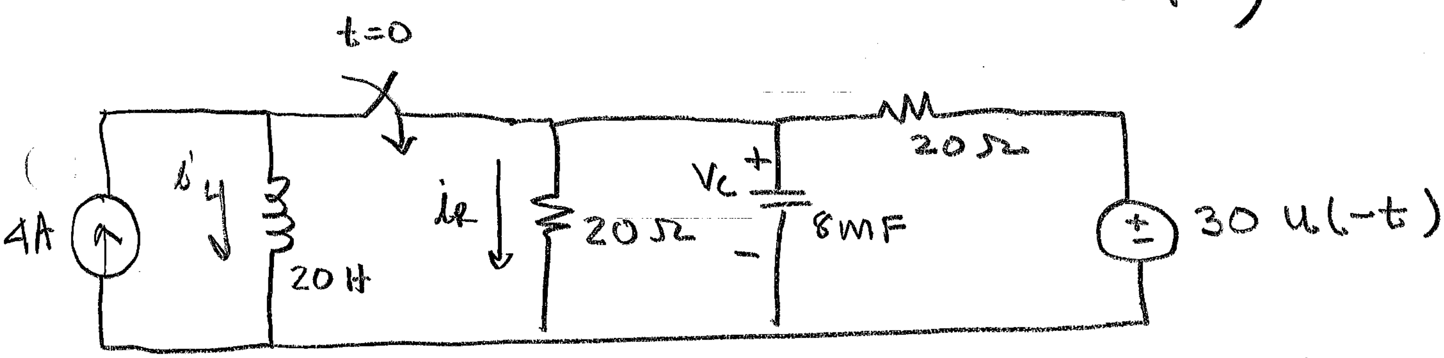
- It's still a parallel RLC

$s_1, s_2 = -11.98\ ,\ -0.5218$

- It is still overdamped.

So we need to

- Get Initial and final conditions, $i_R(0^+)$, $\frac{di_R}{dt}(0^+)$, $i_R(\infty)$



$$i_R(0^+) = \frac{V_R(0^+)}{20} = \frac{V_C(0^+)}{20}$$

and $V_C(0^+) = V_C(0^-) = 30 \times \frac{20}{20+20} = 15\,V$

so $i_R(0^+) = \frac{15}{20} = 0.75\,A$

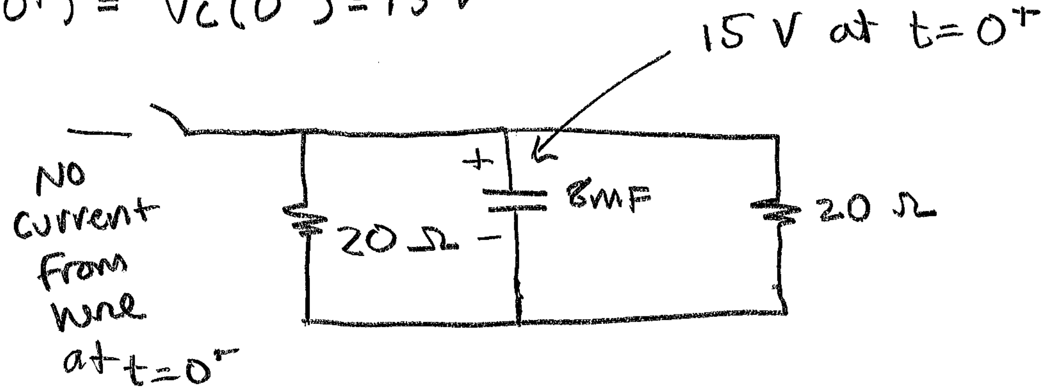
$i_R(\infty) = 0$ because $V_L = V_R$ and $V_L(\infty) = 0$

or I_s

Now get $\frac{di_R(0^+)}{dt} = \frac{1}{R}\,\underbrace{\frac{dV_R(0^+)}{dt} = \frac{1}{R}\,\frac{dV_C(0^+)}{dt}}_{\text{Because R and C are in parallel}}$

$$\frac{dV_c(0^+)}{dt} = \frac{i_c(0^+)}{C}$$

$$V_C(0^+) = V_C(0^-) = 15\,V$$



$$i_c(0^+) = -\frac{15}{20//20} = -\frac{15}{10} = -1.5\,A$$ reasonable sign?

so $\frac{dV_C}{dt}(0^+) = \frac{i_C(0^+)}{C} = \frac{-1.5}{.008} = -187.5\ V/s$

and $d\left(\frac{i_R}{dt}\right) = \frac{1}{R}\,\frac{dV_r(0^+)}{dt} = \frac{1}{R}\,\frac{dV_c(0^+)}{dt}$

$$= \frac{-187.5}{20} = -9.375\,\frac{A}{s}$$

3 things we need

$I_R(0^+) = 0.75A$

$I_R(\infty) = 0$

$\frac{dI_R}{dt}(0^+) = -9.375\,A/s$

$$i_R(0^+) = I_S + A_1 + A_2$$

or $$A_1 + A_2 = i_R(0^+) - I_S = 0.75 - 0$$

and $$\frac{di_R(0^+)}{dt} = -11.98A_1 - 0.5218A_2 = -9.375$$

so $$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -11.98 & -0.5218 \end{bmatrix}^{-1} \times \begin{bmatrix} 0.75 \\ -9.375 \end{bmatrix}$$

and the solution is $A_1 = 0.7840$, $A_2 = -0.0340$

and $$i_R(t) = I_S + A_1 e^{-11.98t} + A_2 e^{-0.5218t}$$

$$i_R = 0 + 0.7840e^{-11.98t} - 0.0340e^{-0.5218t}$$

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we chose to do the problem twice instead of taking a derivative.

Re-consider...

In the first part of the problem we computed

$$i_L(t) = 4 + 0.0655\left[e^{-0.5218t} - e^{-11.98t}\right]$$

we know $V_L(t) = L\dfrac{di_L}{dt}$

and since $V_R = V_L$

$$i_R(t) = \frac{V_L(t)}{R} = \frac{L}{R}\,\frac{d(i_L)}{dt}$$

$$= \frac{\overset{=20\,H}{L}}{\underset{=20\,\Omega}{R}} \times \left(-0.5218 \times 0.0655\, e^{-0.5218t} + 0.0655 \times 11.98 \times e^{-11.98t}\right)$$

$$\underbrace{i_R(t) = -0.0342\, e^{-0.5218t} + 0.7847 e^{-11.98t}}$$

So the derivative was easier !