LECTURE 24 - INTEGRODIFFERENTIAL EQUATIONS AND ACOUSTICS

Integrodifferential equations, Sec 15.6

Practice problem 15.16

Compute $y(t)$ at $t=0$ and $t=\infty$

Compute an expression for $y(t)$

$$\frac{dy}{dt} + 3\,y(t) + 2\int_0^t y(\tau)\,d\tau = 2e^{-3t}, \quad y(0)=0$$

$$sY + 3Y + \frac{2}{s}Y = \frac{2}{s+3}$$

Don't know what Y is but I can solve for it using properties

$$sY(s+3) + 3Y(s+3) + \frac{2}{s}Y(s+3) = 2$$

$$s^2Y + 3sY + 3sY + 9Y + 2Y + \frac{6}{s}Y = 2$$

$$s^3Y + 6s^2Y + 11sY + 6Y = 2s$$

$$\boxed{Y = \frac{2s}{s^3+6s^2+11s+6}}$$

good!

$$\text{Get } y(0) = \lim_{s\to\infty}\left[sY\right] = \frac{2\cdot\infty^2}{\infty^3+\ldots} = 0$$

$$\text{Get } y(\infty) = \lim_{s\to 0}\left[sY\right] = \frac{0}{6} = 0$$

Compute $y(t)$

Factor denominator to identify partial fraction form

$$Y = \frac{2s}{(s+1)(s+2)(s+3)} = \frac{A_1}{s+1} + \frac{A_2}{s+2} + \frac{A_3}{s+3}$$

How did I do this?

"Method of Algebra"

$$A_1(s+2)(s+3) + A_2(s+1)(s+3) + A_3(s+1)(s+2) = 2s$$

$$A_1(s^2+5s+6) + A_2(s^2+4s+3) + A_3(s^2+3s+2) = 2s$$

This will give 3 equations in 3 unknowns . . .
Too much work! (I use residue method for simple poles)

$$A_1 = (s+1)\,Y(s)\Big|_{s=-1} = \frac{2s}{(s+2)(s+3)}\Big|_{s=-1} = \frac{-2}{2} = -1$$

$$A_2 = (s+2)\,Y(s)\Big|_{s=-2} = \frac{2s}{(s+1)(s+3)}\Big|_{s=-2} = \frac{-4}{-1} = 4$$

Similarly, $A_3 = -3$

so

$$Y(s) = \frac{-1}{s+1} + \frac{4}{s+2} - \frac{3}{s+3}$$

$$y(t) = \left(-e^{-t} + 4e^{-2t} - 3e^{-3t}\right)u(t)$$

Check $y(\infty) = 0$, $y(0) = -1+4-3 = 0$.

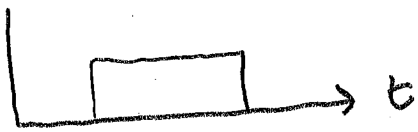
Convolution integral

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda \qquad \text{EQ 15-68b}$$

term that "slides" during convolution.

$x(t)$

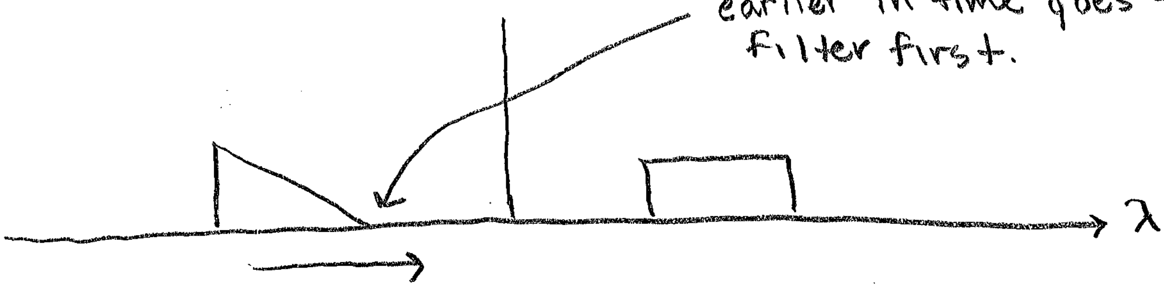
"input signal"



$h(t)$

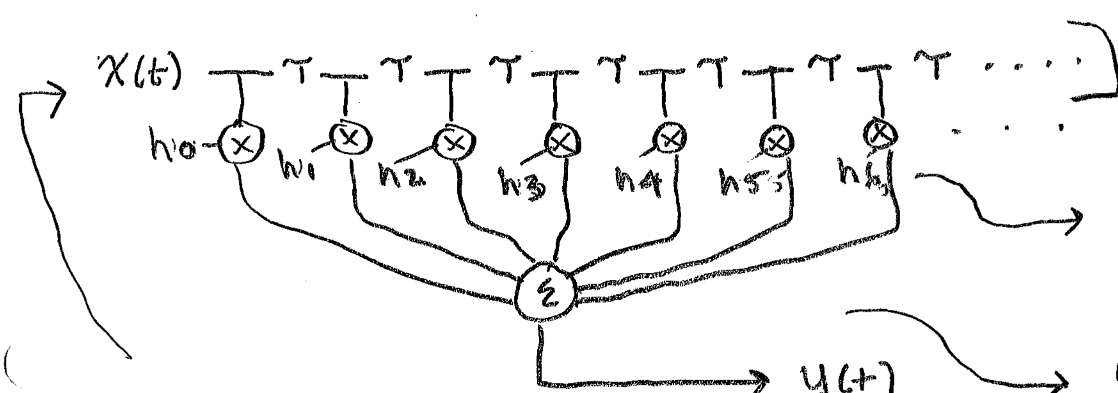
"system impulse response"

earlier in time goes through filter first.



As t changes, $x(t)$ slides past $h(t)$

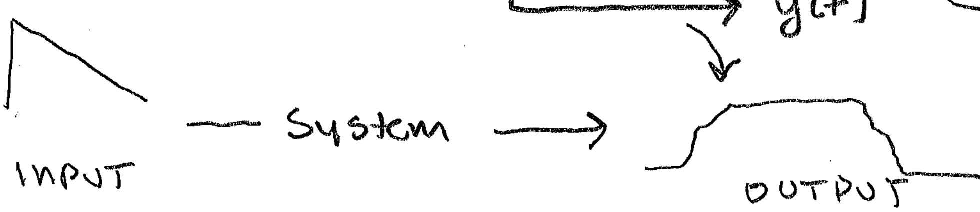
Now let's do the integration as a sum...



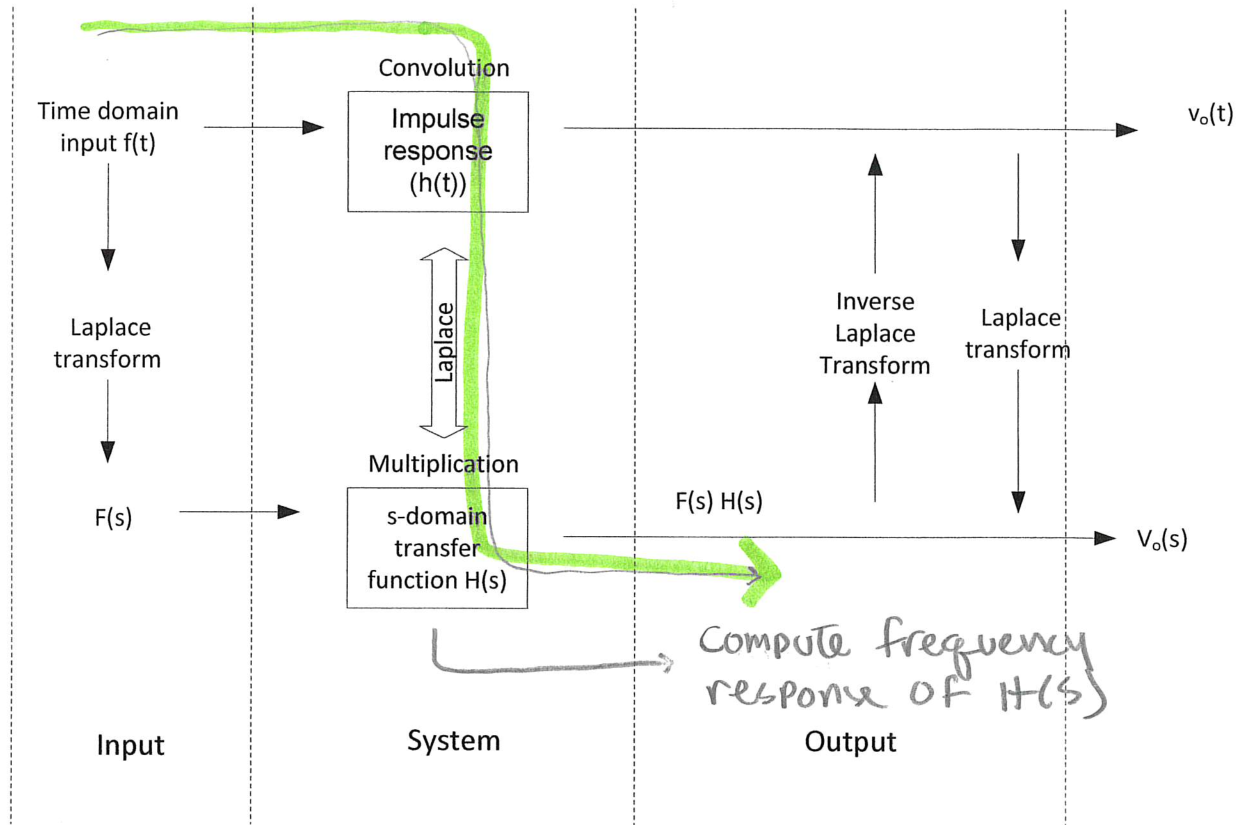
shift register is λ axis

$h_0, h_1 \ldots$ discrete representation of $h(t)$

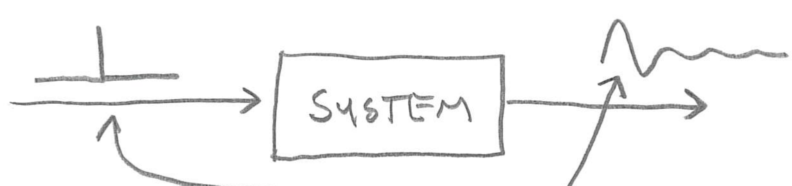
Multiply and sum is a discrete implementation of the integral.



LECTURE 24 - LAPLACE TRANSFORM APPLICATIONS CONTINUED

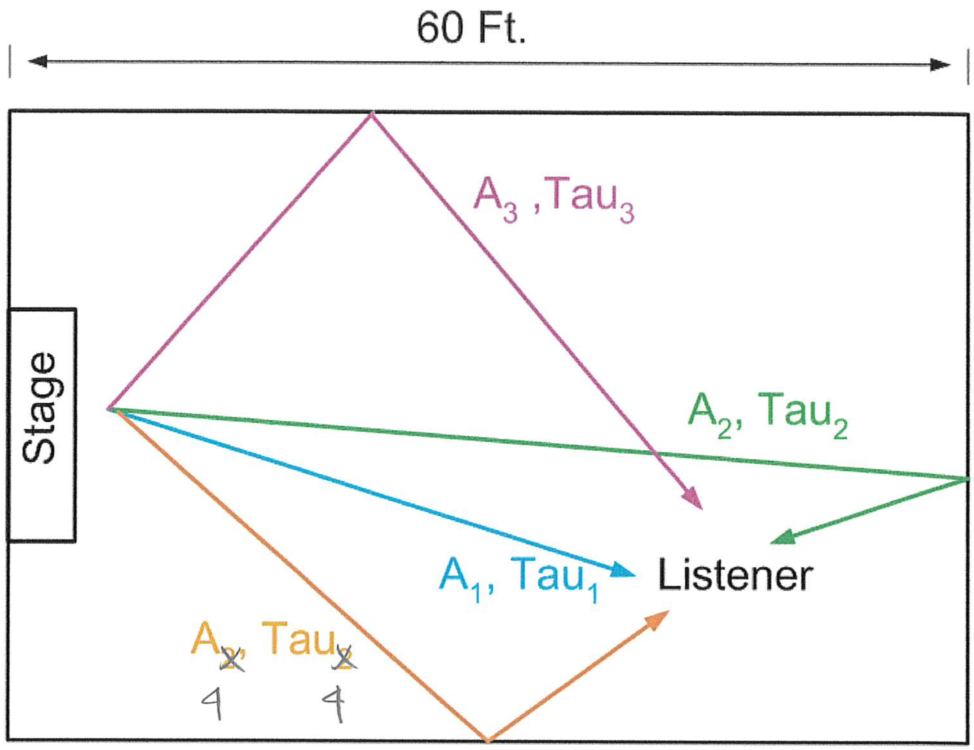


COMPUTE THE Frequency response of a system



1) Feed impulse into system
2) Capture impulse response
3) Compute Laplace xform of impulse response → xfer function.
4) Compute H(s)

Room Acoustics



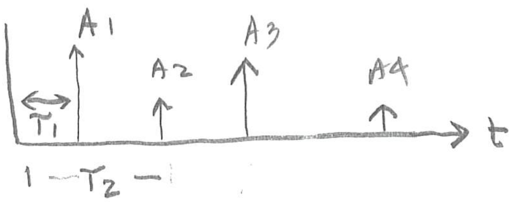
Path	A	T
1	0.5	35.4 ms
2	0.15	46.5 ms
3	0.4	62 ms
4	0.2	71 ms

Given this information

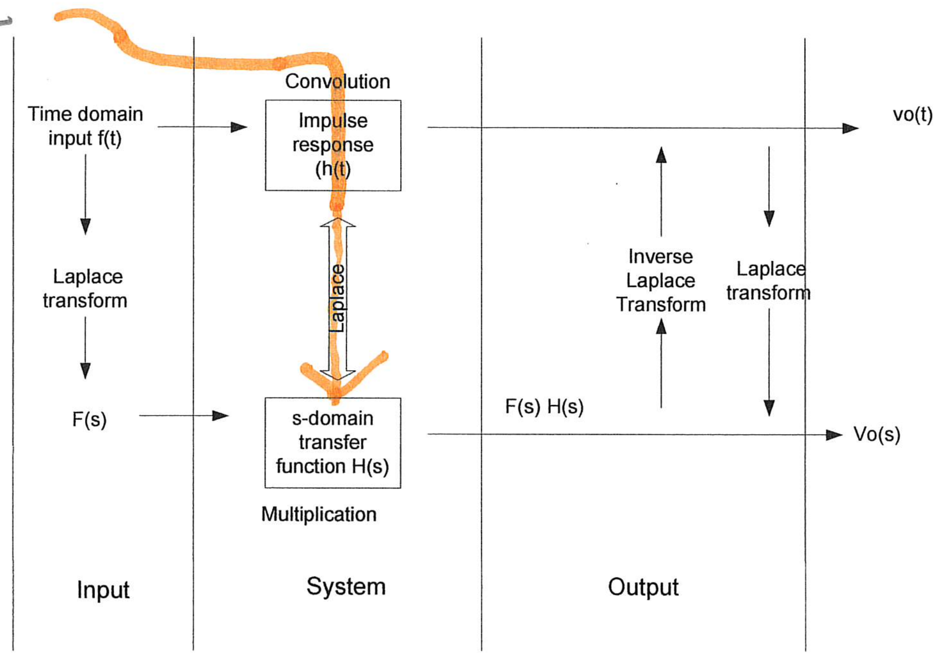
- Compute the frequency response of the room
- Compute the impulse response of the room
- Compute the response of the room to an arbitrary input.

Compute the impulse response of the room by inspection

$$h(t) = A_1 \delta(t - T_1) + A_2 \delta(t - T_2) + A_3 \delta(t - T_3) + A_4 \delta(t - T_4)$$



Impulse

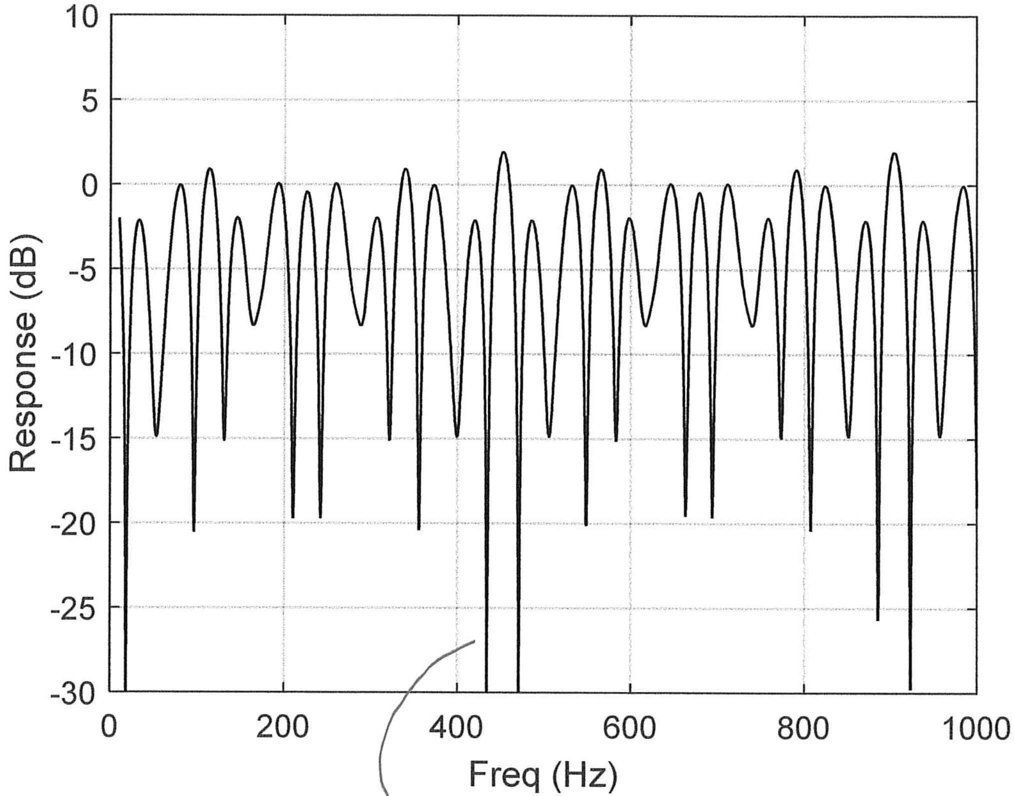


Use delay property to get Laplace xform of impulse response

$$H(s) = A_1 e^{-T_1 s} + A_2 e^{-T_2 s} + A_3 e^{-T_3 s} + A_4 e^{-T_4 s}$$

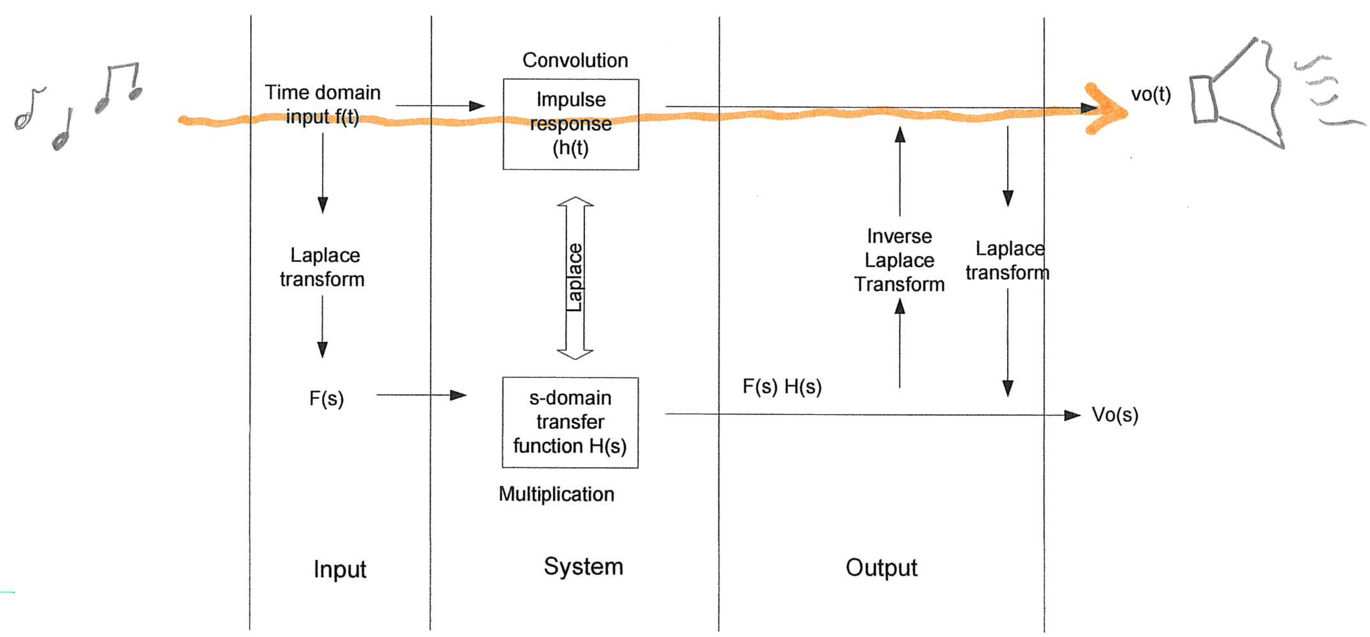
Get Frequency response just like we did in Ch. 14.
Simply substitute $s = j\omega$ into the transfer function

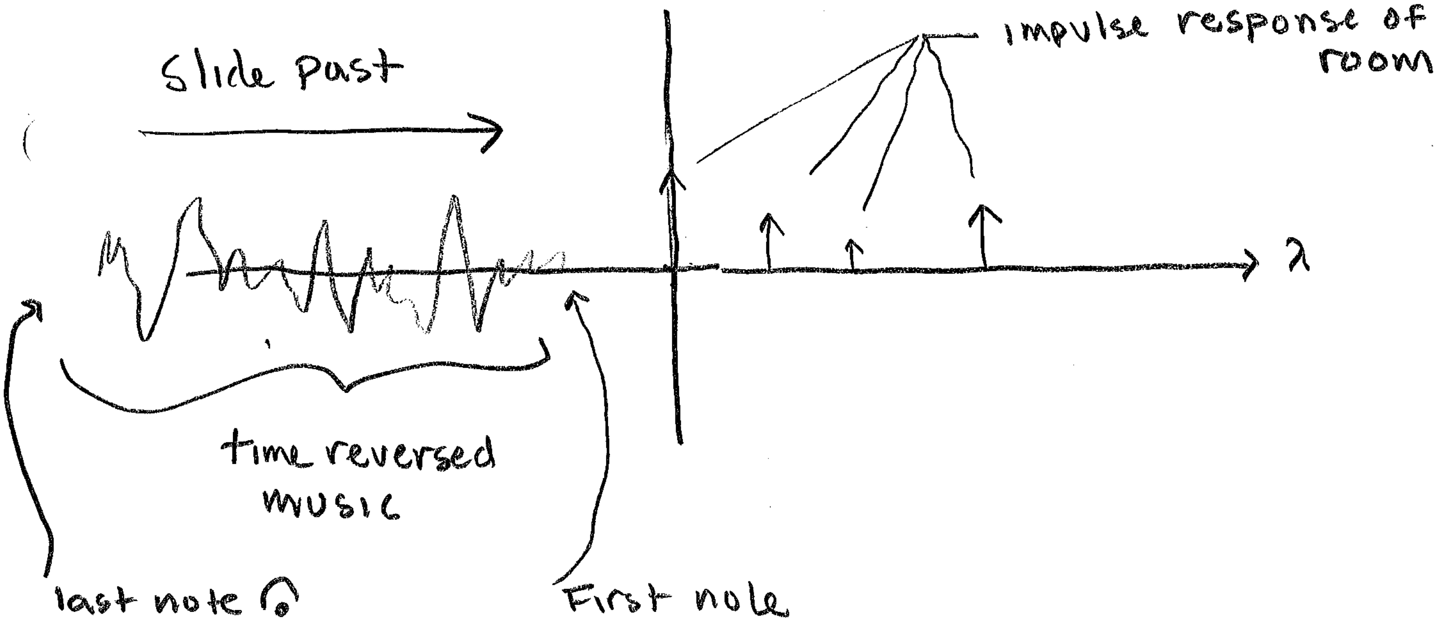
FREQUENCY RESPONSE OF ROOM



IF this is A440, listener wouldn't hear orchestra tuning note!

Now lets "play" a piece of music in this room

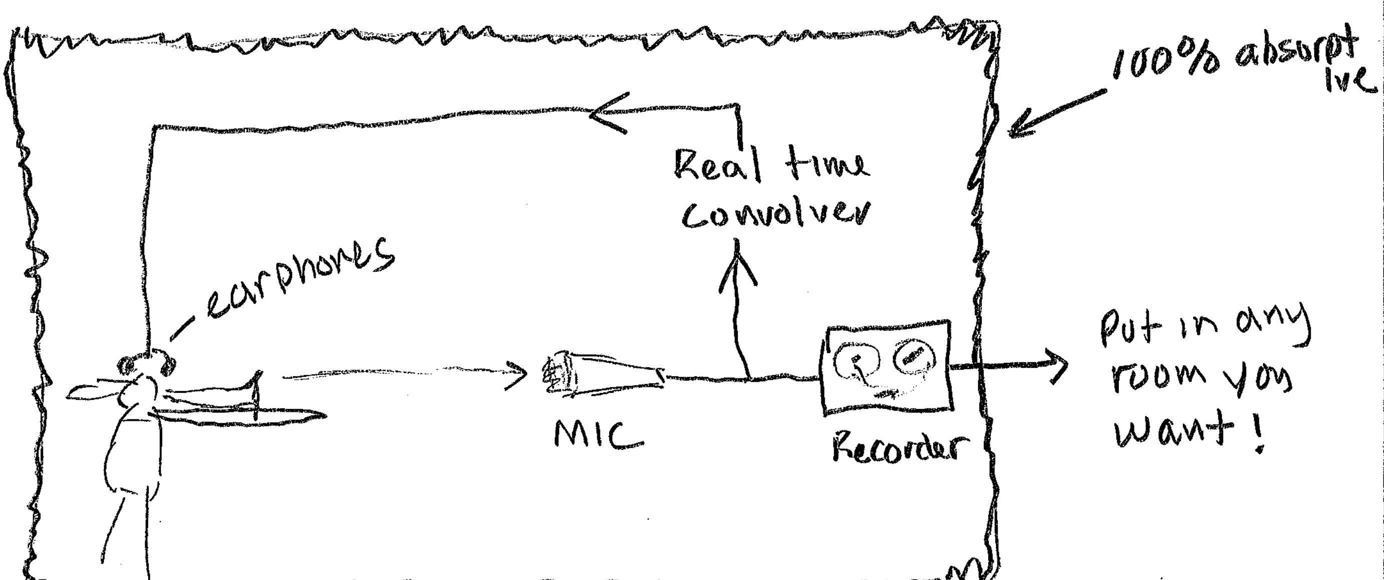




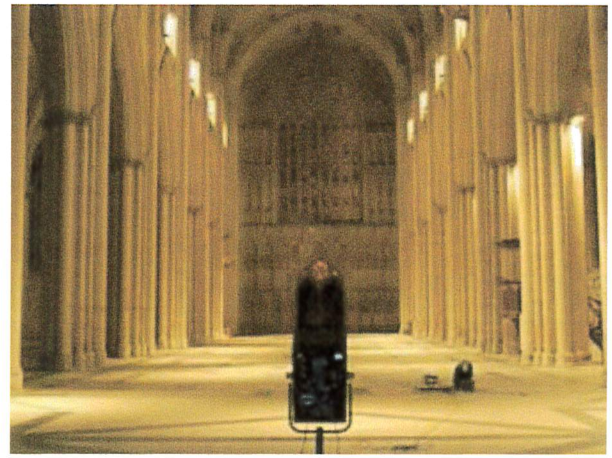
→ What would it sound like if there was only one path (one impulse in response)

→ How do recording studios make "dry" recordings?

→ How does recording studio get a "dry" recording, but fool the performer into thinking he/she is in a big venue?

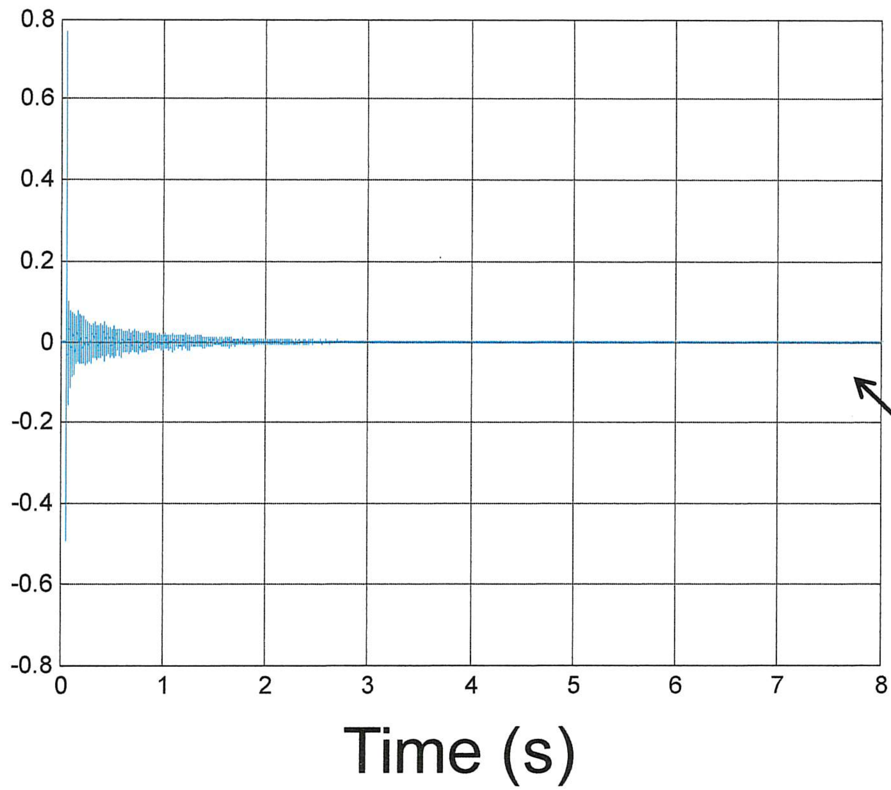


York Minster Cathedral



14th Century

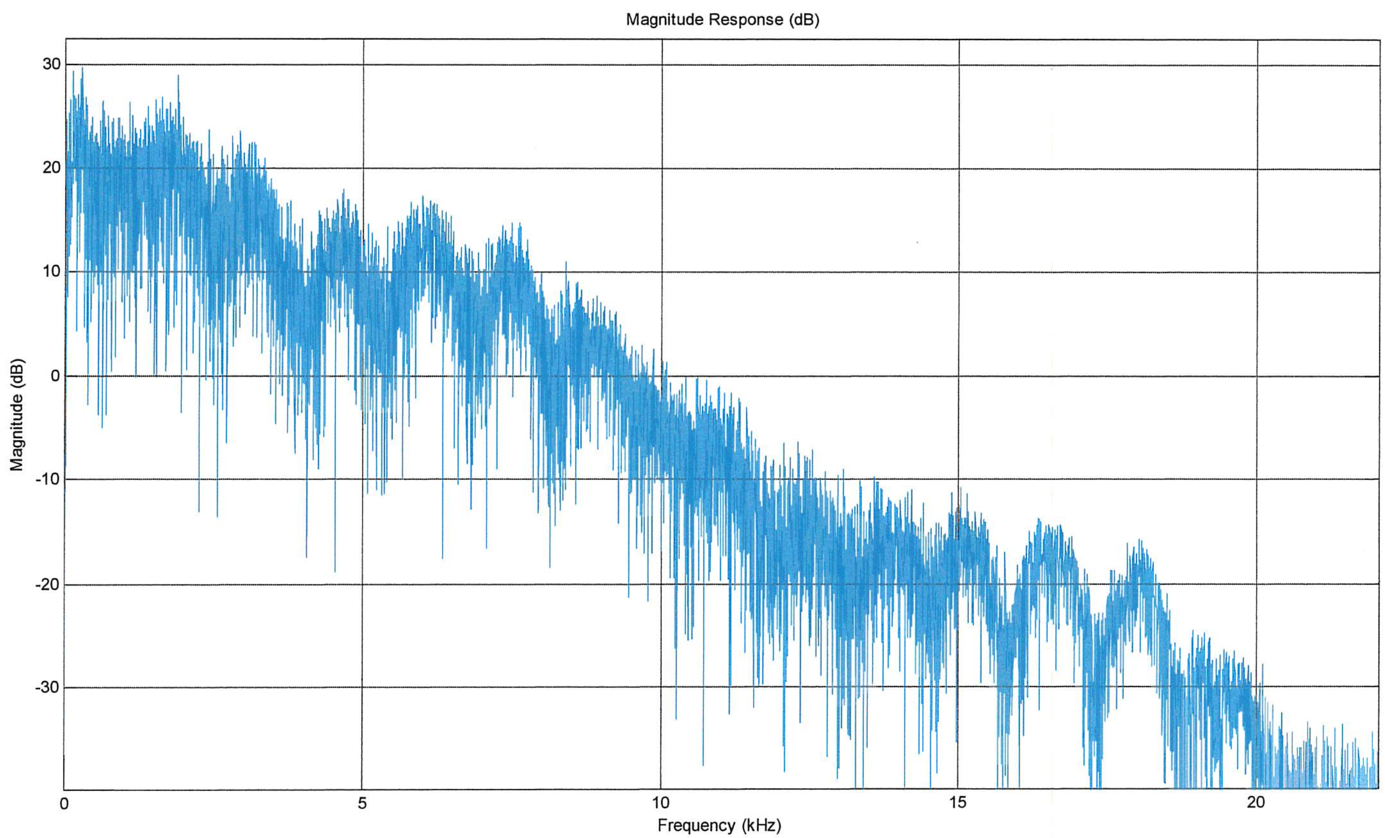
York Minster Impulse Response



Reverberation Time is the time for the sound pressure to decrease by 60 dB after it is stopped.

R.T = 8 seconds)

York Minster Frequency Response



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Different Styles at York Minster

Gabrielli (~1570)

Monteverdi (~1600)

Louis Armstrong (~1955)

Trombone (2015)

What Could Possibly Go Wrong Here?