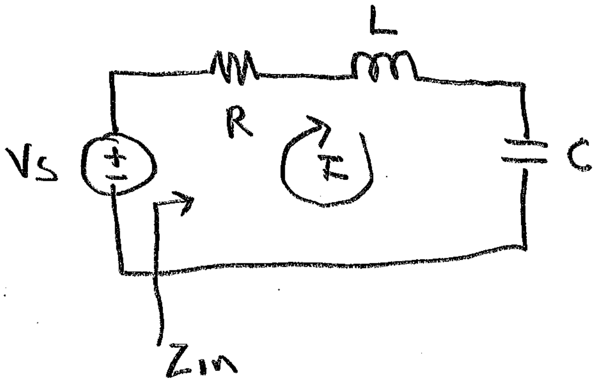
LECTURE 19 SERIES / PARALLEL Resonance and Ladder Networks

"Resonance" is characterized by a sharp peak in the amplitude vs. frequency characteristic.

"Resonance is a condition in an RLC circuit in which the capacitaive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance" } PG 630



$$Z_{in} = Z_R + Z_L + Z_C$$

$$Z_{in} = R + j\omega L + \frac{1}{j\omega C}$$

$$Z_{in} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Set "magnitudes of capacitive and inductive reactance to be equal"

$$\omega L - \frac{1}{\omega C} = 0 \quad \text{then} \quad Z_{in} = R$$

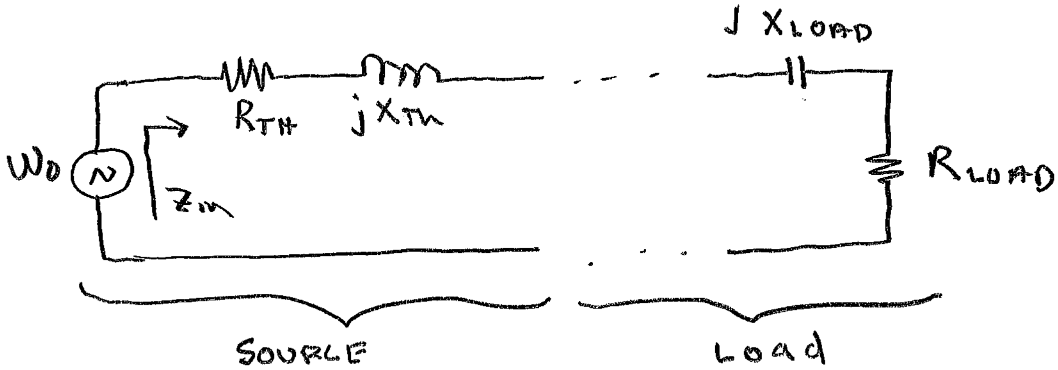
Solve for ω_0.

$$\omega_0 L = \frac{1}{\omega_0 C} \Longrightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

At resonance (pg 630)

1) Input impedance is purely resistive.
2) Voltage and current from supply are in phase
3) The magnitude of Z_{in} is minimum
4) The inductor and capacitor voltages can be much larger than V_s

Relationship of resonance and maximum power transfer,



$Z_{Th} = R_{TH} + jX_{Th}$, $Z_{Load} = R_{LOAD} + jX_{Load}$

for maximum power transfer at ω_0, $Z_{LOAD} = Z_{TH}^*$

Therefore $R_{LOAD} = R_{TH}$ ← Duh..

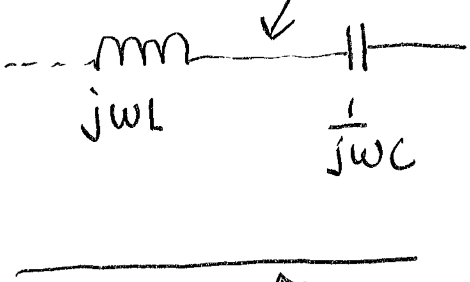
and $X_{LOAD} = -X_{TH}$ — capacitor needed

Since $X_{TH} = \omega L$, $X_{LOAD} = -\omega L = \frac{-1}{\omega C}$

so $\omega_0 L = \frac{1}{\omega C}$ and $C = \frac{1}{\omega^2 \cdot L}$

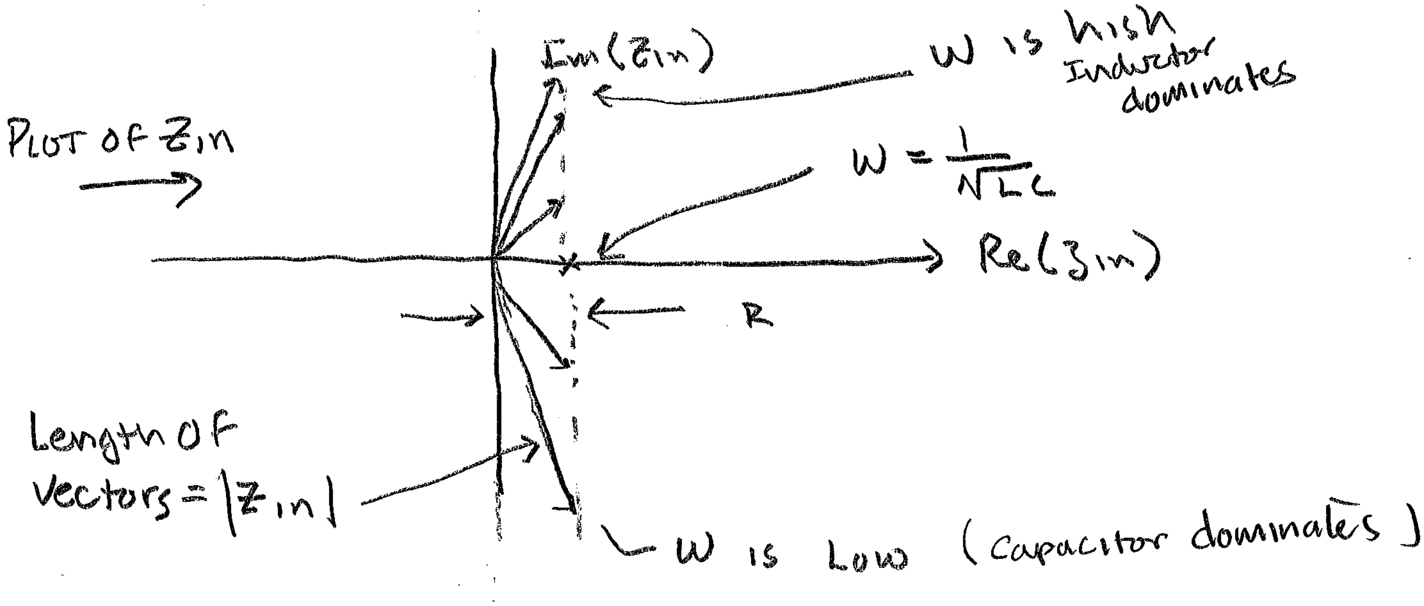
or $\omega_0 = \frac{1}{\sqrt{LC}}$

Big voltage here



$j\omega L + \frac{1}{j\omega C} = 0$

PLOT Z_{in} as seen by voltage source



At $\omega = \omega_0$ Magnitude of Z_{in} is minimized

Since $|I| = \dfrac{|V_{in}|}{|Z_{in}|}$, $|I|$ is maximized at ω_0

Since power in load resistor is $\frac{1}{2}|I|^2 \cdot R$,

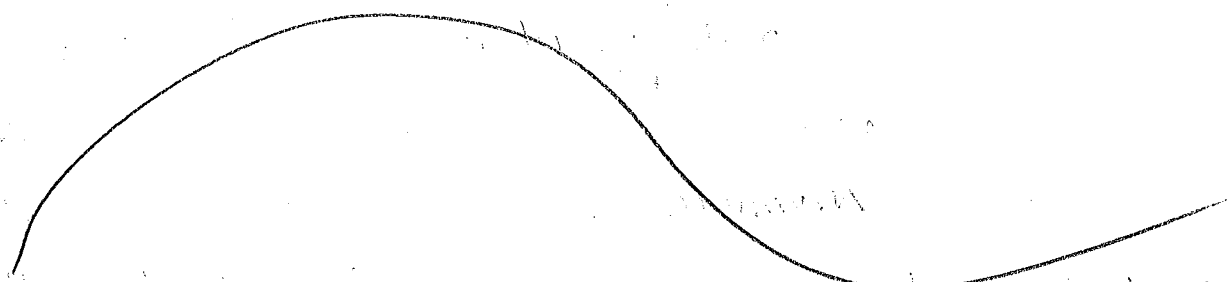
Power in load resistor is maximized at ω_0.

How much power at ω_0 ... or max power xfer?

$$\frac{1}{2}|I|^2 R = \frac{1}{2}\left[\frac{|V|}{2R}\right]^2 \times R = \frac{1}{2}\,\frac{|V|^2}{4R^2} \times R = \frac{|V_{in}|^2}{8R}$$

$I = \dfrac{V}{2R}$

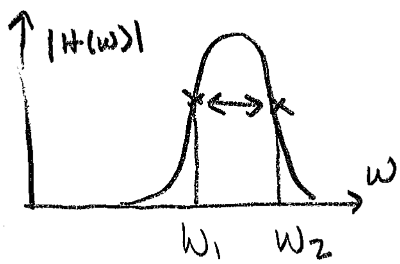
EQ 11.20



Resonant circuits are characterized by their bandwidth.

" How wide is that peak ? "

Define bandwidth as shown below



Bandwidth is $\omega_2 - \omega_1$

ω_2 and ω_1 are frequences where circuit dissipates half the power as it does at resonance.

conceptually important

$P_{resistor} = \frac{1}{2}|I|^2 R$ at resonance

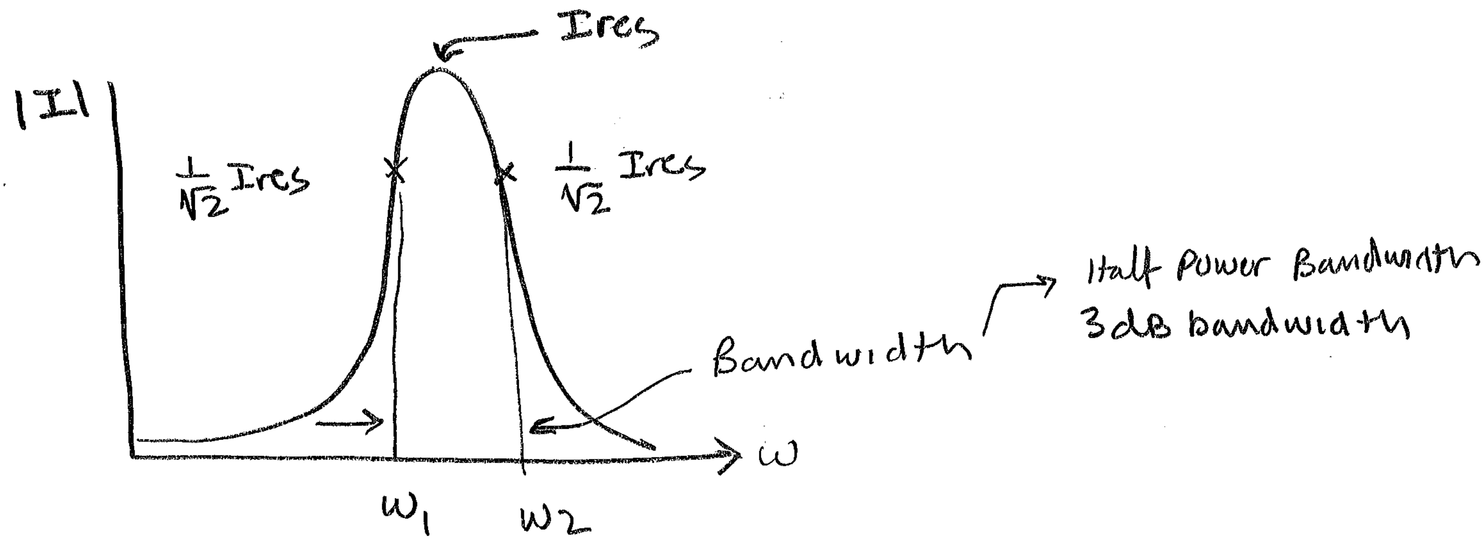
At half power frequencies, $P_{resistor} = \frac{1}{2}\Big[\underbrace{\tfrac{1}{2}|I|^2 R}_{\text{power at resonance}}\Big] = \frac{1}{2}\Big[\frac{I}{\sqrt{2}}\Big]^2 R$

So ω_1, ω_2 are frequences where $I = \frac{1}{\sqrt{2}} I_{resonance} = \frac{1}{\sqrt{2}} \frac{V_{in}}{R}$

ω_1, ω_2 are also called the "3 dB frequencies"

or the "half-power frequencies"

From this we get the "half power" frequencies



$$\left.\begin{aligned} \omega_1 &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\ \omega_2 &= \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \end{aligned}\right\} \quad \text{EQ 14.33}$$

$$\omega_1 \omega_2 = \left(\frac{R}{2L}\right)^2 + \frac{1}{LC} - \frac{R^2}{4L^2} = \frac{1}{LC}$$

So $\frac{1}{\sqrt{LC}} = \sqrt{\omega_1 \omega_2} = \omega_0$ EQ 14.34

and we see ω_0 is the geometric mean of the half-power frequencies

The "half power bandwidth" is

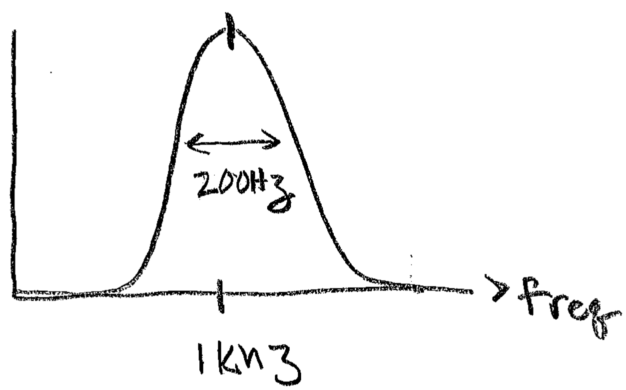
$$B = W_2 - W_1 = \frac{R}{2L} + \sqrt{\quad} - \left[-\frac{R}{2L} + \sqrt{\quad} \right]$$

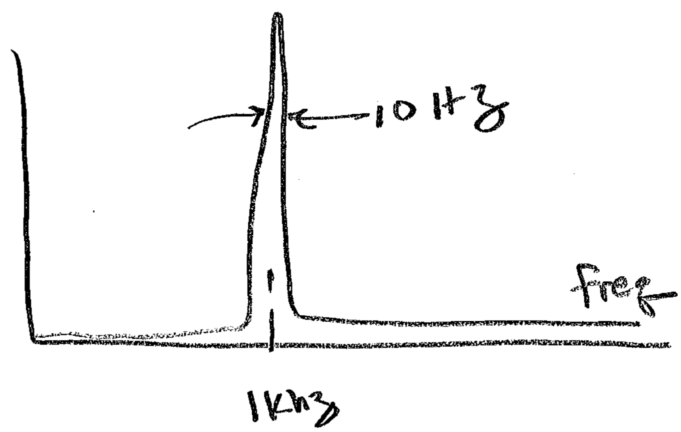
$$B = \frac{R}{L}$$

"Quality factor" $= Q = \frac{W_0 L}{R} = \frac{1}{W_0 RC}$ EQ 14.38

OR $B = \frac{W_0}{Q}$ EQ 14.39

Bandwidth in r/s $= \frac{W_0}{Q}$, Bandwidth in Hz $= \frac{f_0}{Q}$





$Q = \frac{1\,\text{kHz}}{200\,\text{Hz}} = 5$

"Low Q"

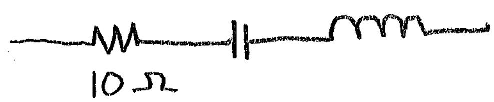
$Q = \frac{1\,\text{kHz}}{10\,\text{Hz}} = 100$

"High Q"

Which filter is "more selective?"

PR 14.28

Design a series resonant circuit with $BW = 20 \, r/s$ and $\omega_0 = 1000 \, r/s$. Find Q. Let $R = 10\,\Omega$



$$BW = \underbrace{20 \, r/s = \frac{R}{L}}_{\text{use}} = \frac{\omega_0}{Q} \qquad \text{EQ 14.38}$$

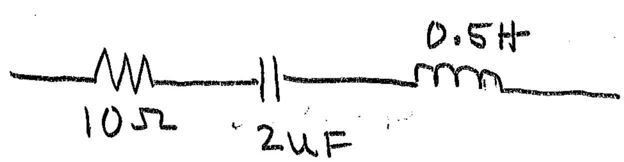
so $L = \dfrac{R}{20} = \dfrac{10}{20} = 1/2 \; H$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \text{EQ 14.26}$$

so $\omega_0^2 LC = 1$ or $C = \dfrac{1}{\omega_0^2 L} = \dfrac{1}{1000^2 \times 0.5}$

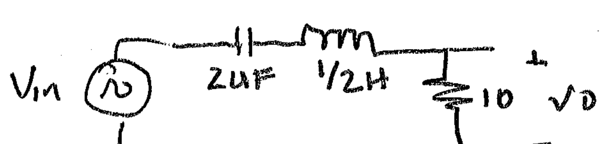
$= 2 \, \mu F$

and $Q = \dfrac{\omega_0 L}{R} = \dfrac{1000 \times 0.5}{10} = 50$



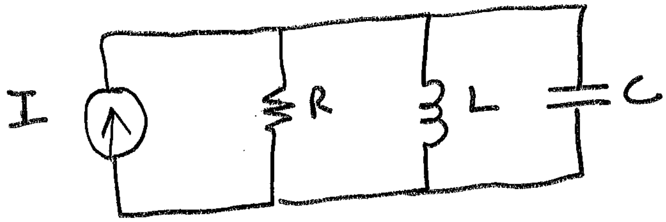
check...

$\omega_0 = \dfrac{1}{\sqrt{LC}} = 1000 \, r/s \qquad BW = \dfrac{R}{L} = 20 \, r/s \qquad Q = \dfrac{\omega_0}{BW} = \dfrac{1000}{20} = 50$



What is $\dfrac{V_o}{V_{in}}$ at $\omega = \omega_0$?

Parallel resonance



At resonance $Y_C = -Y_L$

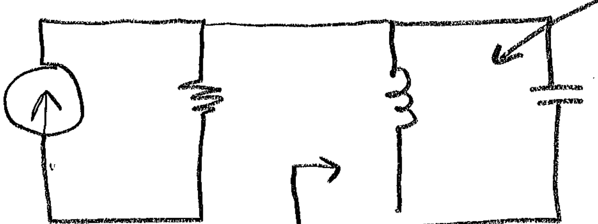
cap, ind

or $j\omega_0 C = -\frac{1}{j\omega_0 L} = \frac{j}{\omega_0 L}$

solving $\omega_0 C = \frac{1}{\omega_0 L}$ or $\omega_0 = \frac{1}{\sqrt{LC}}$

Since $Y_c + Y_L = 0$, impedance of parallel combination is $\frac{1}{Y_T} = \frac{1}{0} = \infty$

Big currents in here.



$Z_{in} = \infty$

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}, \quad \omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

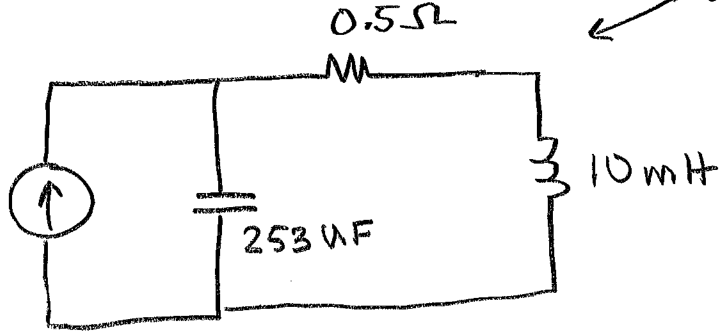
$$\underbrace{BW = \omega_2 - \omega_1 = \frac{1}{RC}}_{\text{Paralell is different than series!}}, \quad Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

Example

what kind of circuit is this ?

I_s

$6\angle 0$



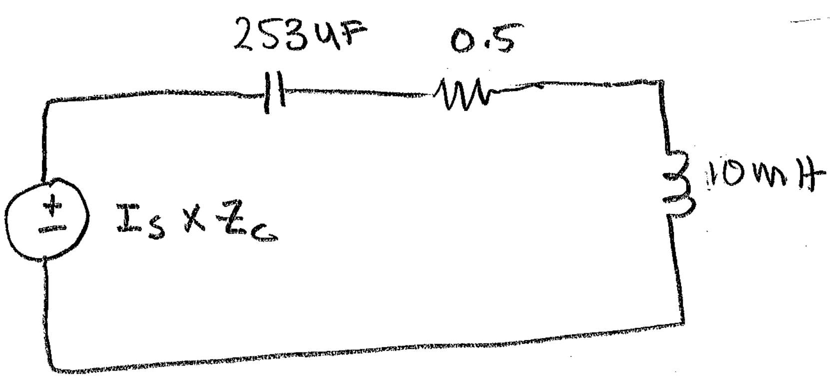
Find resonant frequency in Hz

Find bandwidth in Hz

find Q

find power dissipated in 0.5Ω at resonance

find power dissipated in 0.5Ω at lower 3 dB frequency



$$f_0 = \frac{1}{2\pi} \times \frac{1}{\sqrt{LC}} = 100\,Hz \qquad \text{EQ 14.26}$$

$$BW = \frac{1}{2\pi}\ \frac{R}{L} = \frac{1}{2\pi} \times \frac{0.5}{0.01} = 7.96\,Hz \qquad \text{EQ 14.39}$$

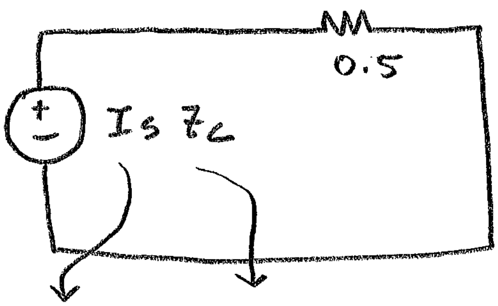
$$Q = \frac{f_0}{BW} = \frac{100}{7.96} = 12.5$$

r/s or Hz as long as they are the same

CONT →

Find power dissipated in the 0.5 Ω resistor at resonance

At resonance $Z_C + Z_L = 0$ so current is computed using the equivalent circuit



$$6\angle 0 \times \frac{1}{j2\pi \times 100 \times 253\,\mu F} = -j37.74\ V$$

↑ resonant frequency

$$P = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2} \times \frac{37.74^2}{0.5} = 1424.3\ W$$

Find the power dissipated in the 0.5 Ω resistor at the lower 3 dB frequency.

- Use EQ 4.31 for series resonance

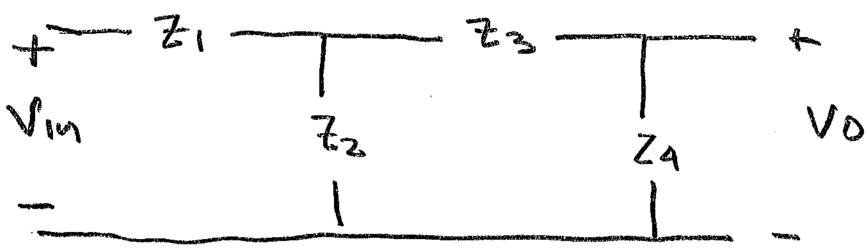
$$P = \frac{V_m^2}{4R} = \frac{37.74^2}{4 \times 0.5} = 712.1\ W$$

- OR note that at the 3 dB (half power) frequences the current is $1/\sqrt{2}$ times the peak current and

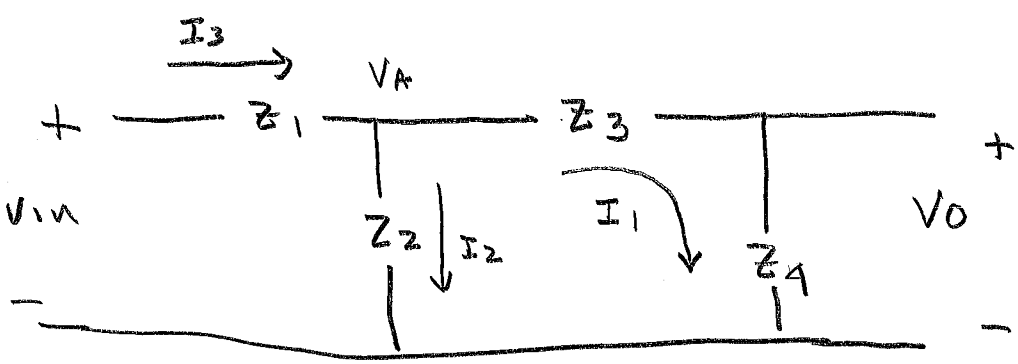
$$\frac{P_{3dB}}{P_{res}} = \frac{\frac{1}{2}\left|\frac{1}{\sqrt{2}} I_m\right|^2 R}{\frac{1}{2}|I_m|^2 R} = \frac{1}{2}$$

LADDER NETWORKS

Find V_O/V_{in}



Strategies ?



SET OUTPUT TO $1\angle 0^\circ$ V, compute input then $\dfrac{V_O}{V_{in}} = \dfrac{1\angle 0^\circ}{V_{in}(s)}$

$$I_1 = \frac{1}{Z_4}$$

$$V_A = 1 + I_1 Z_3$$

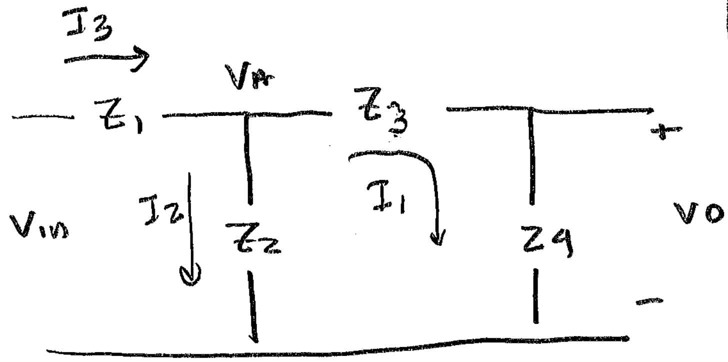
$$I_2 = \frac{V_A}{Z_2}$$

$$I_3 = I_1 + I_2$$

$$V_{in} = V_A + I_3 Z_1$$

Compute, Substitute, Compute . . . .

$$\longrightarrow \frac{V_O}{V_{in}} = \frac{1\angle 0}{V_{in}}$$



$$I_1 = \frac{1}{Z_4}$$

$$V_A = 1 + I_1 Z_3 = 1 + \frac{Z_3}{Z_4}$$

$$I_2 = \frac{V_A}{Z_2} = \frac{1 + \frac{Z_3}{Z_4}}{Z_2}$$

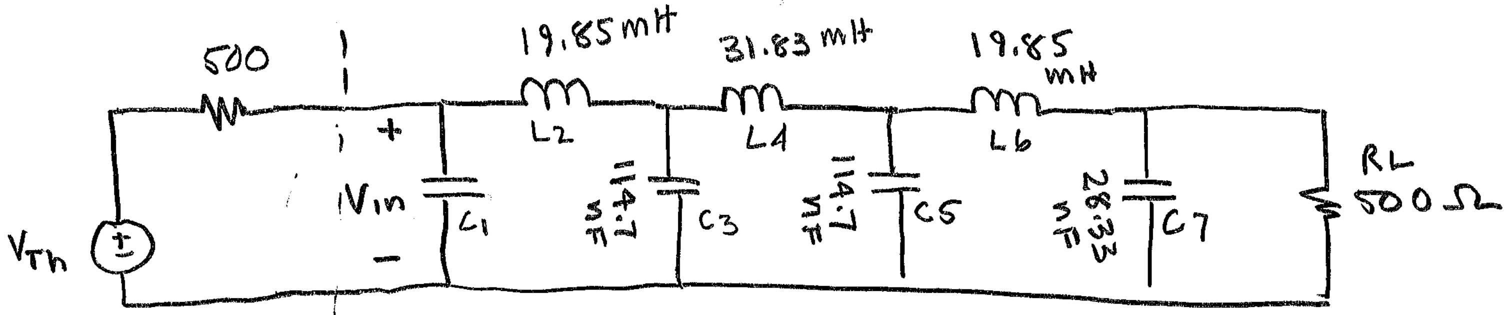
$$I_3 = I_1 + I_2 = \frac{1}{Z_4} + \frac{1 + \frac{Z_3}{Z_4}}{Z_2}$$

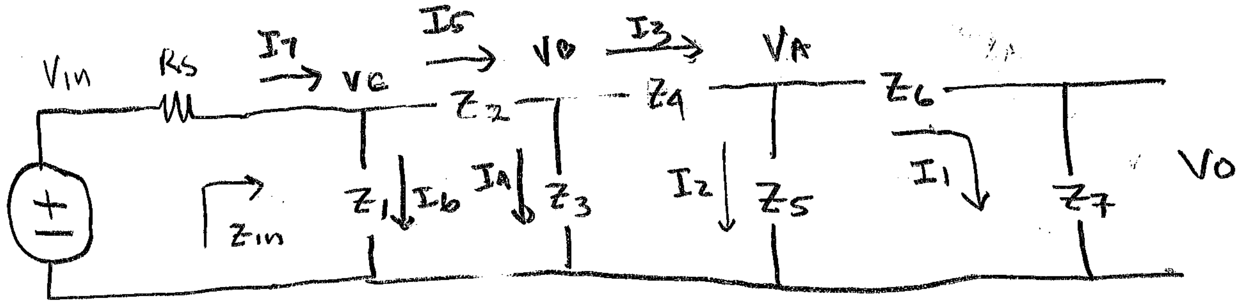
$$V_{in} = V_A + I_3 Z_1 = \underbrace{1 + \frac{Z_3}{Z_4}}_{V_A} + \underbrace{\left[\frac{1}{Z_4} + \frac{1 + \frac{Z_3}{Z_4}}{Z_2}\right]}_{I_3} Z_1$$

$$\frac{V_o}{V_{in}} = \frac{1}{1 + \frac{Z_3}{Z_4} + \left[\frac{1}{Z_4} + \frac{1 + \frac{Z_3}{Z_4}}{Z_2}\right] Z_1}$$

What's so easy about this?

EXAMPLE


500
19.85 mH
31.83 mH
19.85 mH
L2
L4
L6
+
Vin
−
C1
114.7 nF
C3
114.7 nF
C5
28.33 nF
C7
RL
500 Ω
VTh


Vin
Rs
I7
VC
I5
VB
I3
VA
Z2
Z4
Z6
Z1
I6
IA
Z3
I2
Z5
I1
Z7
Vo
Zin

```
clear all;
close all;

% Specify components.
Rs = 500;
C1 = 28.33e-9;
C3 = 114.7e-9;
C5 = 114.7e-9;
C7 = 28.33e-9;
L2 = 19.85e-3;
L4 = 31.83e-3;
L6 = 19.85e-3;
Rl = 500;

StartFreq = 10;      % Lowest frequency to plot.
NumDec = 4;          % Number of decades to plot.
PtsPerDec = 200;     % Number of frequency points plotted per decade.

% Pre-allocate results matricies.
MagResp = zeros(NumDec*PtsPerDec,1);    % Matrix containing magnitude response.
Freq = zeros(NumDec*PtsPerDec,1);       % Matrix containing plot frequencies.


% Compute frequency response at frequency points uniformly-spaced on a log plot.
for i=1 : NumDec*PtsPerDec
    Freq(i) = StartFreq*10^(i/PtsPerDec);    % Evaluate at this frequency.
    s = 1j * 2*pi*Freq(i);                   % Determine complex frequency.

    % Compute branch impedances at this frequency.
    Z1 = 1/(s*C1);
    Z2 = s*L2;
    Z3 = 1/(s*C3);
    Z4 = s*L4;
    Z5 = 1/(s*C5);
    Z6 = s*L6;
    Z7 =  (1/(s*C7) * Rl) / (1/(s*C7) + Rl);

    Vo = 1;
    I1 = 1/Z7;
    Va = 1 + I1*Z6;
    I2 = Va/Z5;
    I3 = I1 + I2;
    Vb = Va + I3*Z4;
    I4 = Vb/Z3;
    I5 = I3 + I4;
    Vc = Vb + I5*Z2;
    I6 = Vc/Z1;
    I7 = I5 + I6;
    Vin = Vc + I7 * Rs;

    H = Vo/Vin;
```

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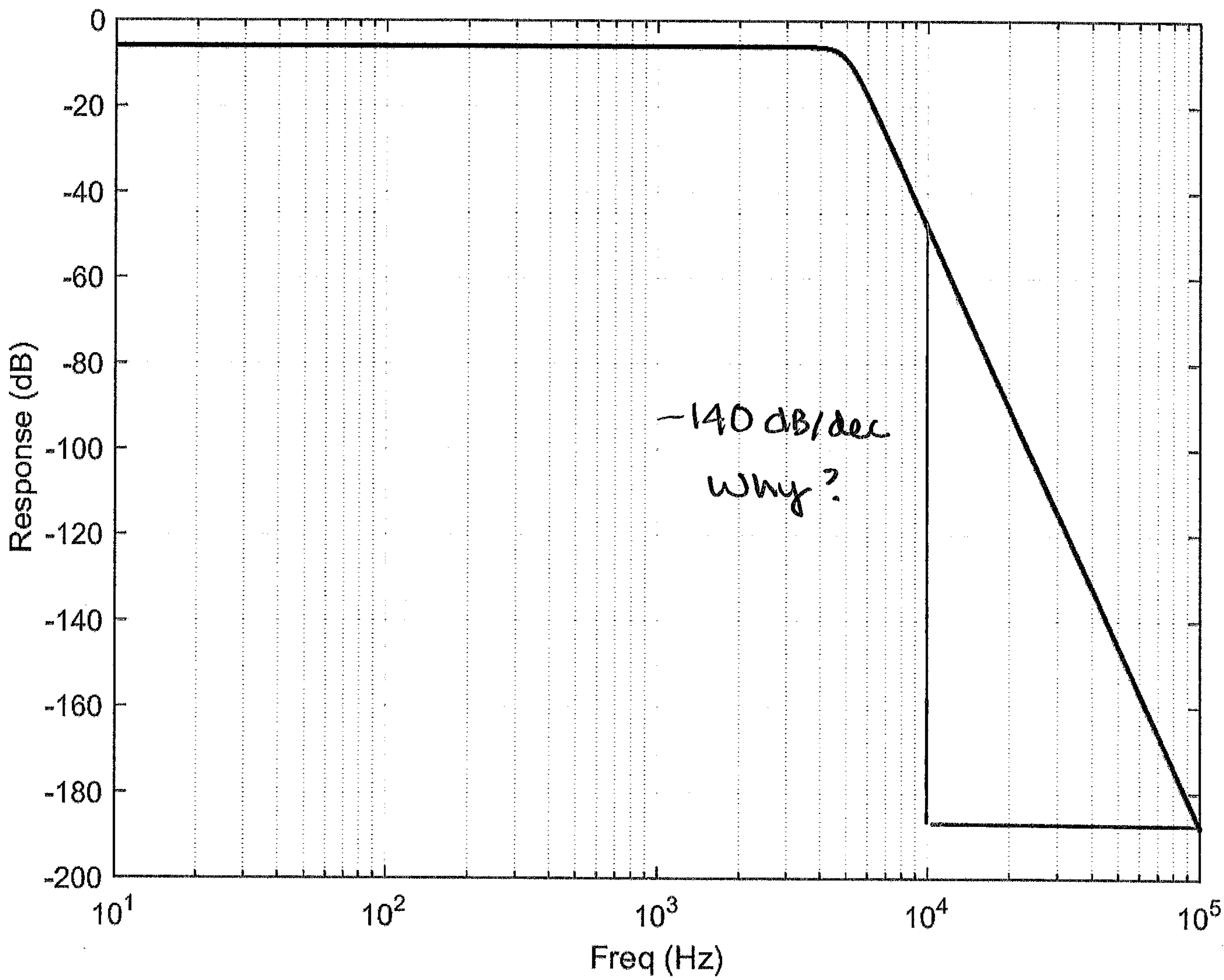
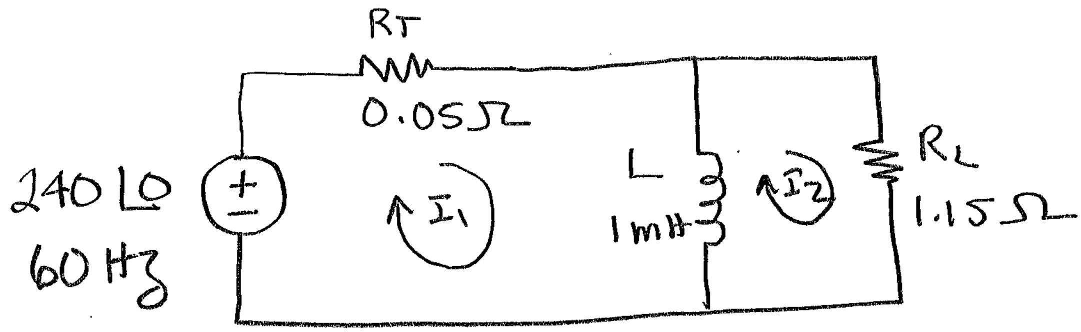
```
    MagResp(i) = abs(H);     % Place magnitude response in result matrix.
end

% Plot magnitude.
HMag=semilogx(Freq,20*log10(MagResp), 'color','k', 'linewidth', 2); % Plot magnitude

set(gca,'FontSize',12);
xlabel('Freq (Hz)');
ylabel('Response (dB)');
% ylim([-50, 10]);
grid on;
```

L13


0
-20
-40
-60
-80
-100
-120
-140
-160
-180
-200
Response (dB)
10^1
10^2
10^3
10^4
10^5
Freq (Hz)
-140 dB/dec
Why?

EXAMPLE



Determine power delivered to R_T and R_L

Strategy: Use mesh analysis to get currents in R_T and R_L

$$P_{WR}(R_T) = \frac{1}{2} I_{1m}^2 R_T$$
$$P_{WR}(R_L) = \frac{1}{2} I_{2m}^2 R_L$$

① $-240 + I_1 R_T + (I_1 - I_2) j\omega L = 0$

$$(R_T + j\omega L) I_1 - j\omega L I_2 = 240$$

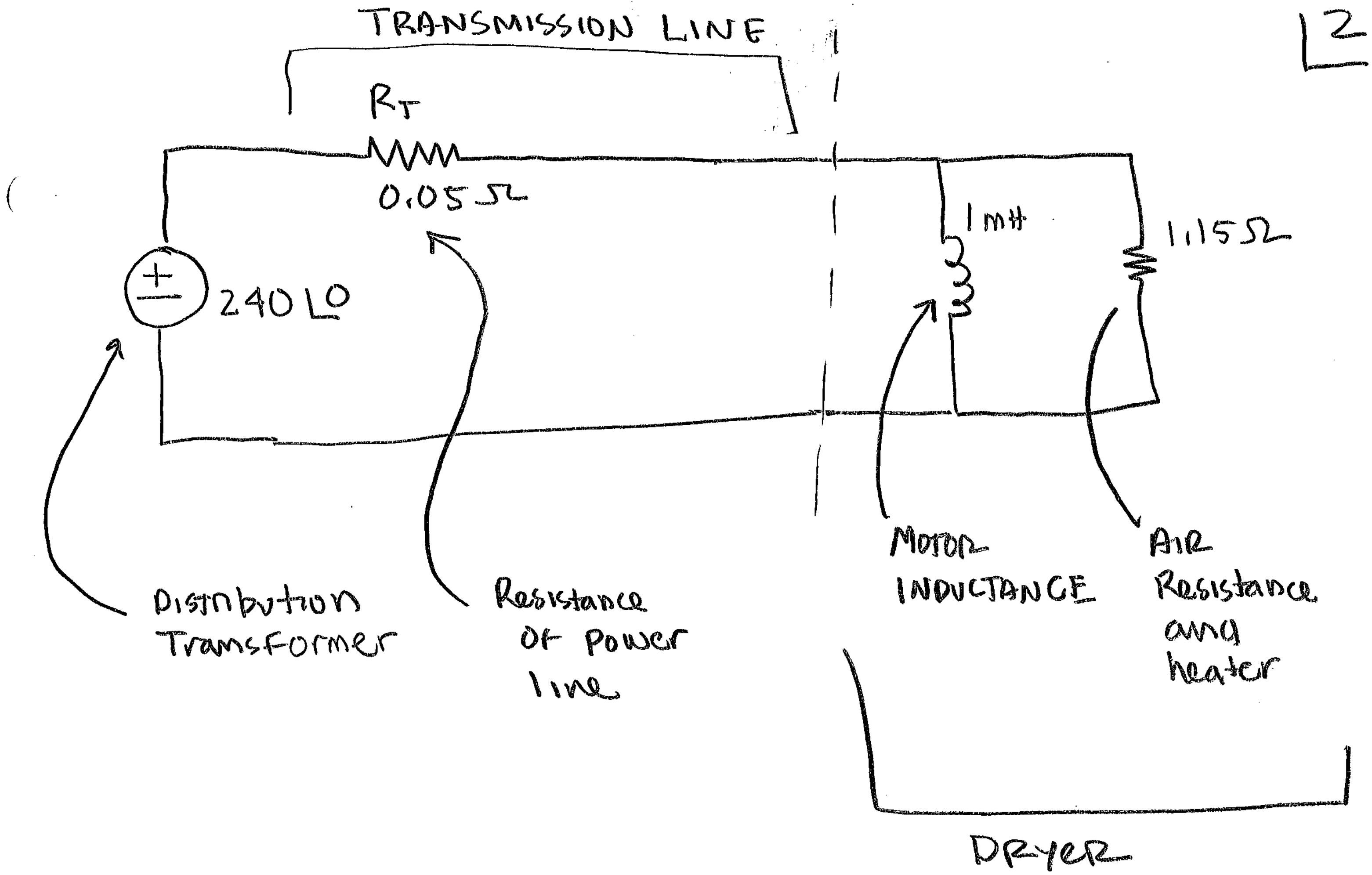
② $(I_2 - I_1) j\omega L + I_2 R_L = 0$

$$-j\omega L \cdot I_1 + (j\omega L + R_L) I_2 = 0$$

$$\begin{bmatrix} R_T + j\omega L & -j\omega L \\ -j\omega L & j\omega L + R_L \end{bmatrix}^{-1} \begin{bmatrix} 240 \\ 0 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{matrix} 636.9 \angle -64.6^\circ \\ 198.4 \angle 7.24 \end{matrix}$$

So $P(R_T) = \frac{1}{2} |I_1|^2 \times R_T = \frac{1}{2} \times (636.9)^2 \times 0.05 = 10.14\ \text{KW}$

$$P_{R_L} = \frac{1}{2} |I_2|^2 \times R_L = \frac{1}{2} (198.4)^2 \times 1.15 = 22.63\ \text{KW}$$

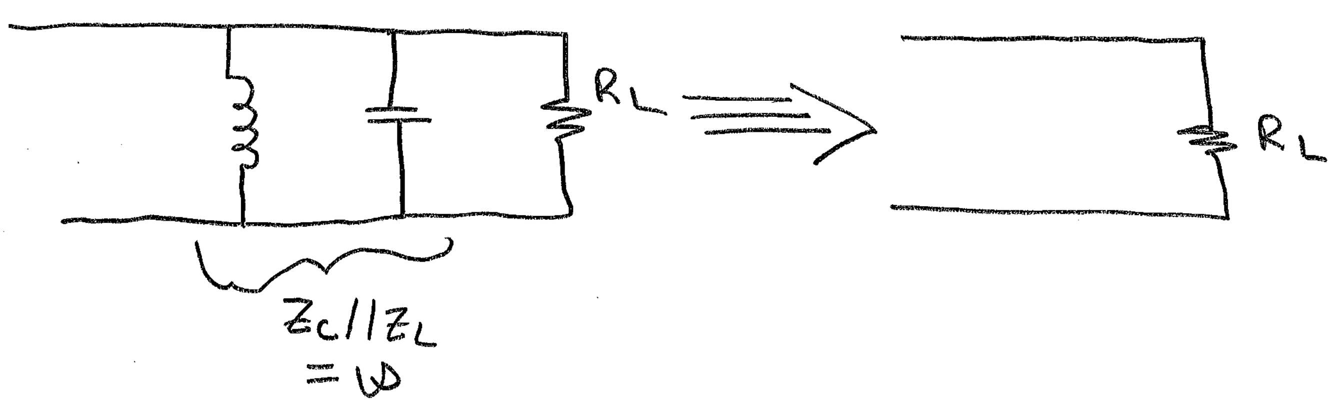


The power company pays for power in R_T

The customer pays for the power in R_L

So the power company places a capacitor at the car wash.

Pick capacitor so $Z_C \| Z_L = \infty$



Set $Z_C // Z_L$ to ∞

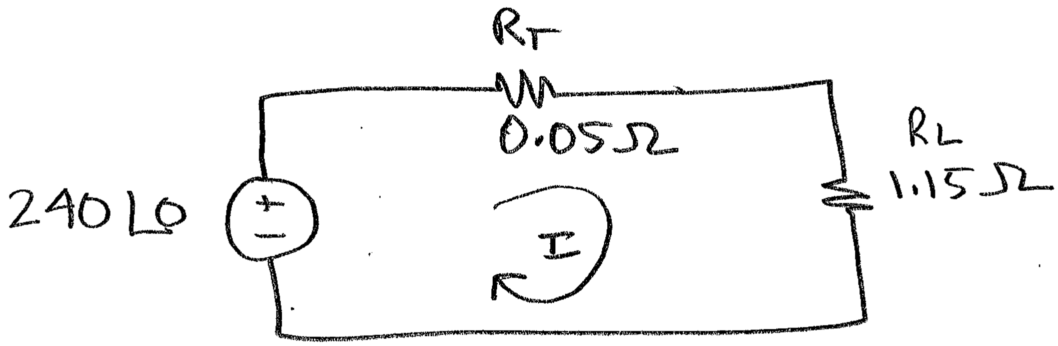
$$\frac{\frac{1}{j\omega C} \times j\omega L}{\frac{1}{j\omega C} + j\omega L} = \frac{L/C}{\underbrace{\frac{-j}{\omega C} + j\omega L}} = \infty$$

Den must be zero so

$$\frac{1}{\omega C} = \omega L \quad \text{or} \quad C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi \times 60)^2 \times 0.001}$$

$$= 7.036\ \mu F$$

Now we have



$$I = \frac{240}{0.05 + 1.15} = 200\,A$$

$$P_{RT} = \frac{1}{2}|I|^2 R_T = \frac{1}{2} \times 200^2 \times 0.05 = 1000\ W$$

$$P_{RL} = \frac{1}{2}|I|^2 R_L = \frac{1}{2} \times 200^2 \times 1.15 = 23.0\ kW$$

	TOTAL	R_T	R_L	% of total in R_L
NO CORR	32.8 KW	10.14 KW	22.63	69%
CORR	24.0 KW	1 KW	23.0 KW	96%

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What's going on?

1) Inductive part of load is an energy storage device.
2) Inductor accepts energy from source, then throws it back.
3) Whenever energy goes across power line, some is lost in the resistance R_T. Power company must pay!
4) The parallel capacitor makes the load site purely resistive (Energy is transferred between inductor and capacitor)

"Power factor Correction - EE 310 style