LECTURE 18 - BODE PLOTS II

Frequency response plots have two components
- Magnitude vs Frequency
- Phase vs Frequency.

Since we plot magnitude in dB, we can <u>add</u> the contributions from each factor.

But how do we plot phase?

$$H(s) = \frac{K \cdot j\omega \cdot \left(1 + \frac{j\omega}{z_1}\right)\left(1 + \frac{j\omega}{z_2}\right)}{\left(1 + j\omega/p_1\right)\left(1 + j\omega/p_2\right)}$$

$$H(s) = \frac{K \cdot \omega \angle 90 \cdot |T_1(s)| \angle T_1(s) \cdot |T_2(s)| \angle T_2(s)}{|T_3(s)| \angle T_3 \cdot |T_4(s)| \angle T_4}$$

$$H(s) = \underbrace{\frac{K \cdot \omega\, |T_1(s)| \cdot |T_2(s)|}{|T_3(s)| \cdot |T_4(s)|}}_{\text{Magnitude}} \; \underbrace{\angle\, 90^\circ + \angle T_1(s) + \angle T_2(s) - \angle T_3(s) - \angle T_4(s)}_{\text{Phase angle}}$$

- The phase angles add
- We can deal with each term independently.

Factor $K \longrightarrow$ Phase angle is zero

Factor $j\omega \longrightarrow$ Phase angle is 90°

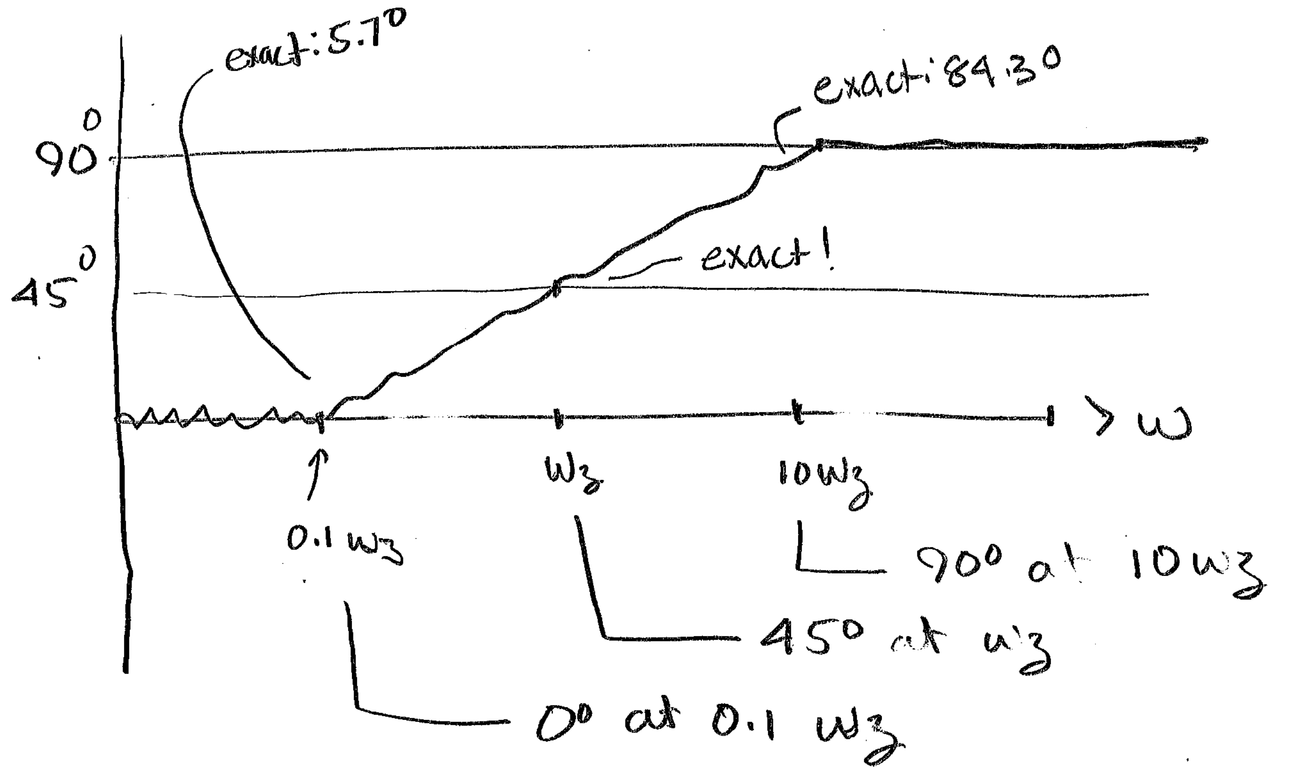
factor $\frac{1}{j\omega} \longrightarrow$ Phase angle is -90

Factor $\left(1 + j\frac{\omega}{\omega_z}\right)$

$\phi = \tan^{-1}\left(\frac{\omega}{\omega_z}\right)$

$\omega = 0 \longrightarrow \tan^{-1}(0) = 0^\circ$
$\omega = \omega_z \longrightarrow \tan^{-1}(1) = 45^\circ$
$\omega = \infty \longrightarrow \tan^{-1}(\infty) = 90^\circ$

Rule



Practice problem 14.4 – Phase plot

$$H(j\omega) = \frac{50}{4\times10^2} \frac{j\omega}{(1+j\omega/4)(1+j\omega/10)^2}$$

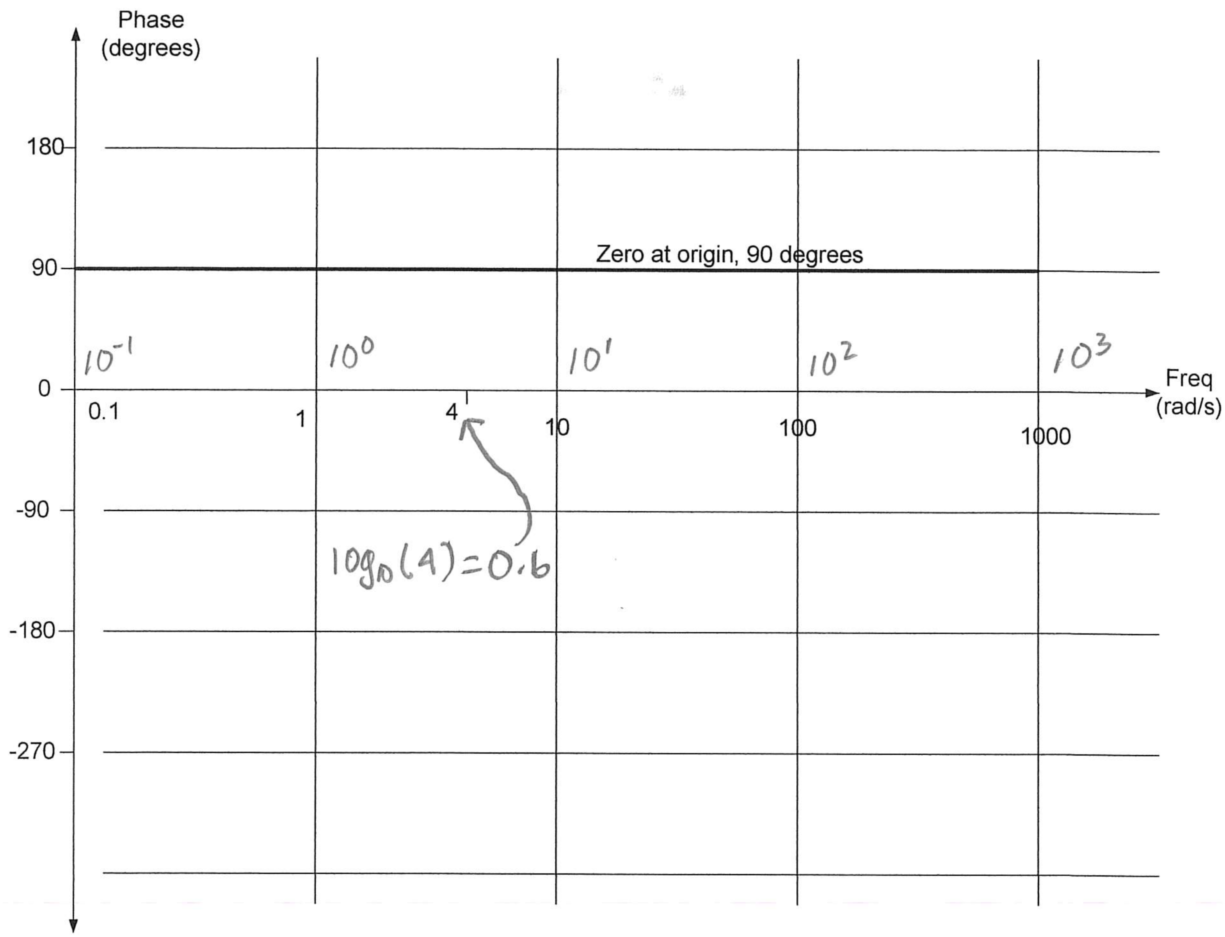


Figure 1 - Zero at Origin

$$H(s) = \frac{1}{8} \cdot \frac{j \cdot \omega}{\left(1 + \frac{j \cdot \omega}{4}\right) \cdot \left(1 + \frac{j \cdot \omega}{10}\right)^2}$$

LS

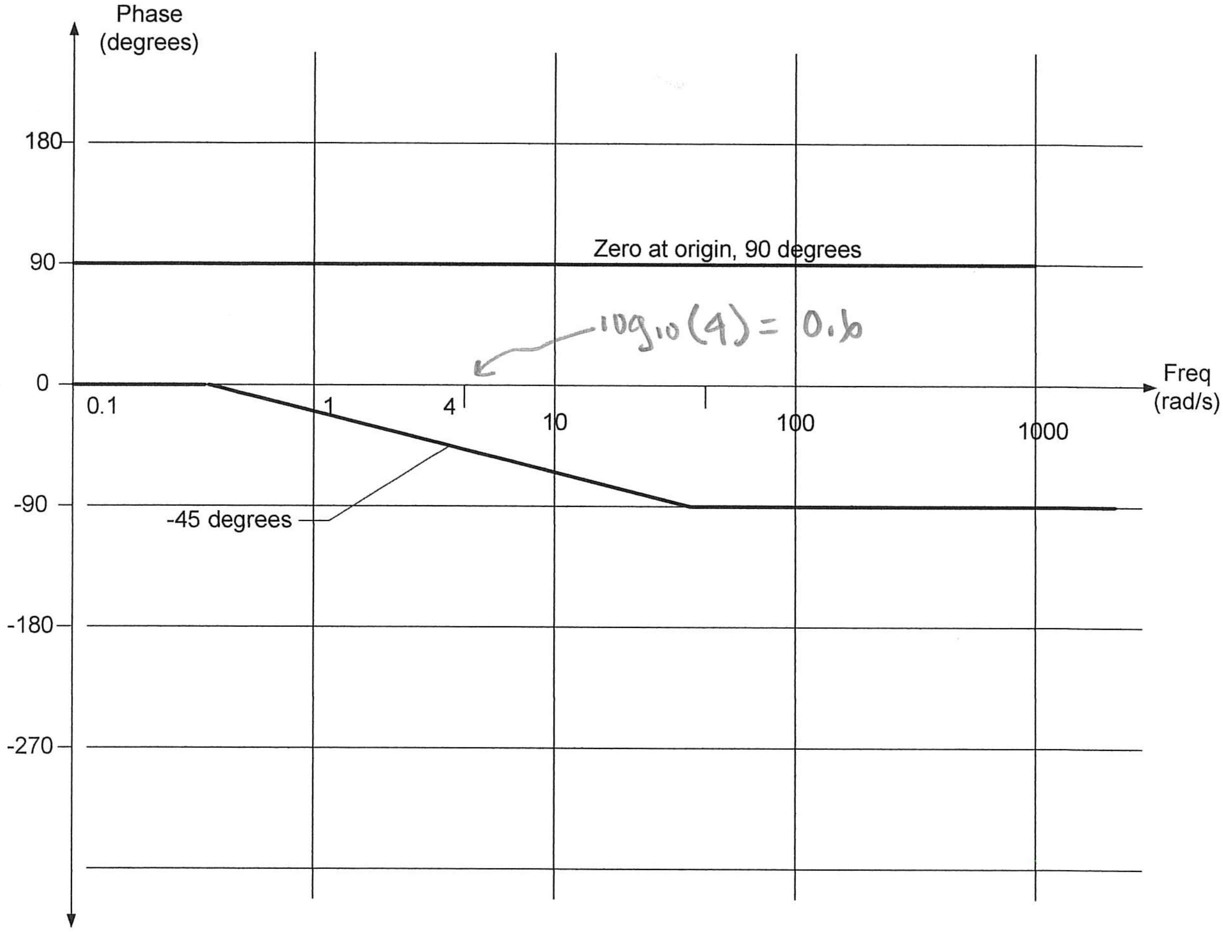


Figure 2 - Pole at 4 rad/s

$$H(s) = \frac{1}{8} \cdot \frac{j \cdot \omega}{\left(1 + \frac{j \cdot \omega}{4}\right) \cdot \left(1 + \frac{j \cdot \omega}{10}\right)^2}$$

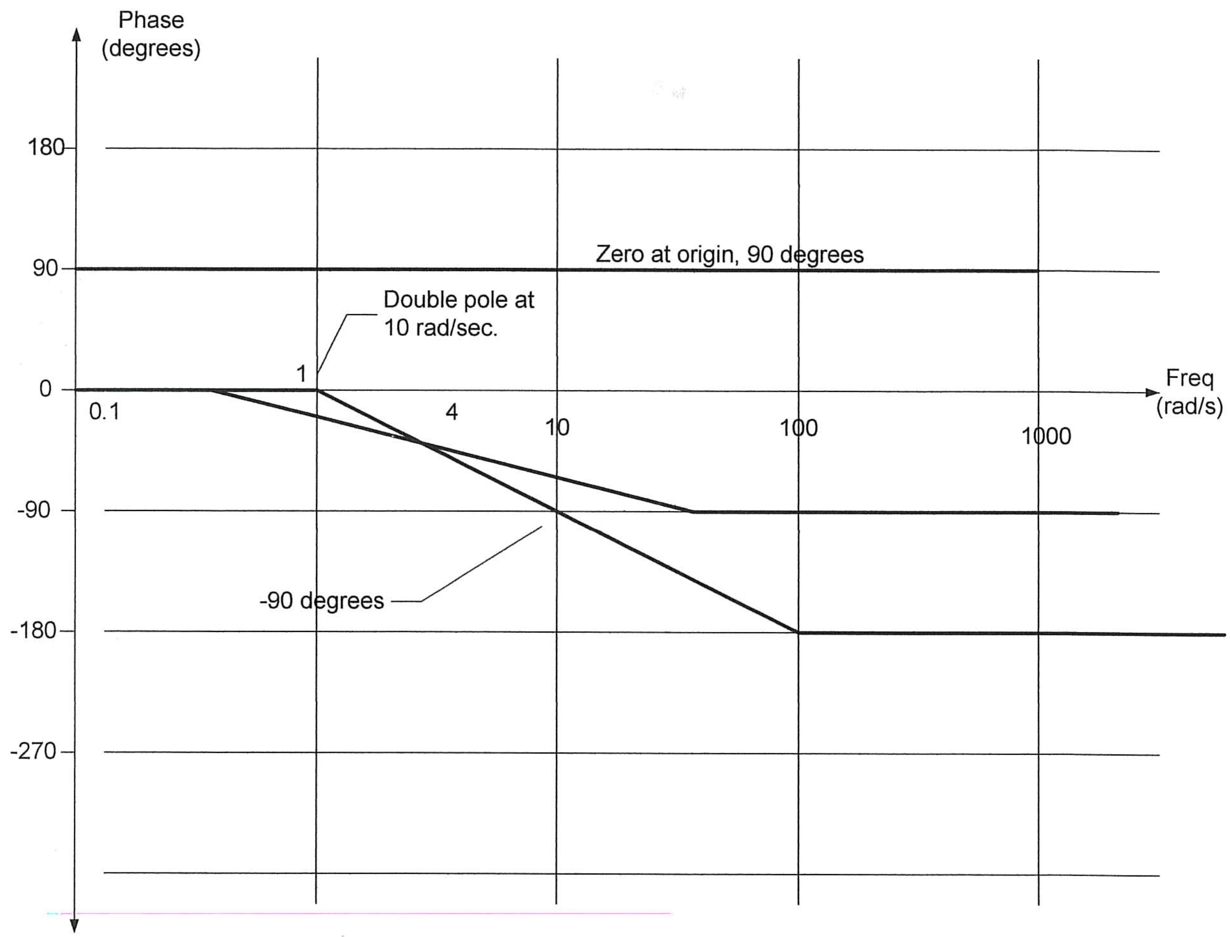


Figure 3 - Double pole at 10 r/s

$$H(s) = \frac{1}{8} \cdot \frac{j \cdot \omega}{\left(1 + \frac{j \cdot \omega}{4}\right) \cdot \left(1 + \frac{j \cdot \omega}{10}\right)^2}$$

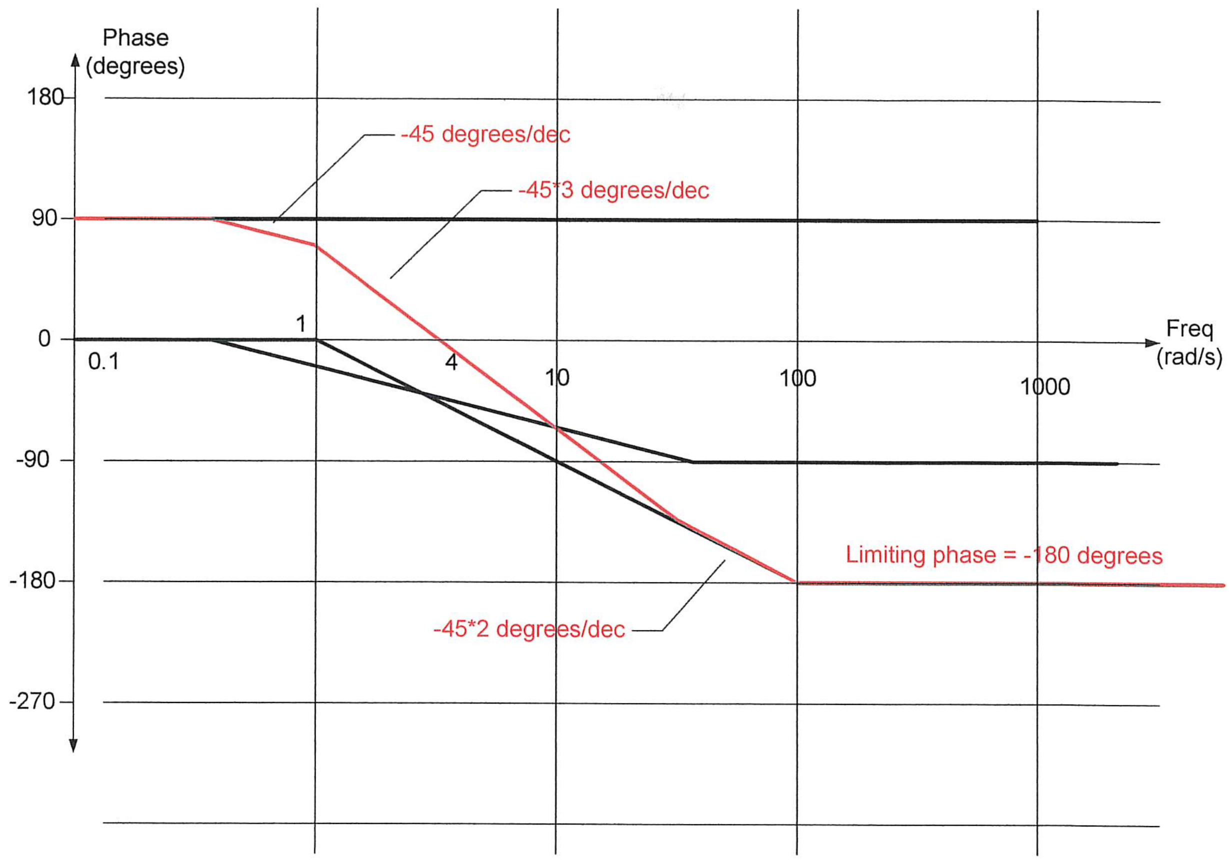
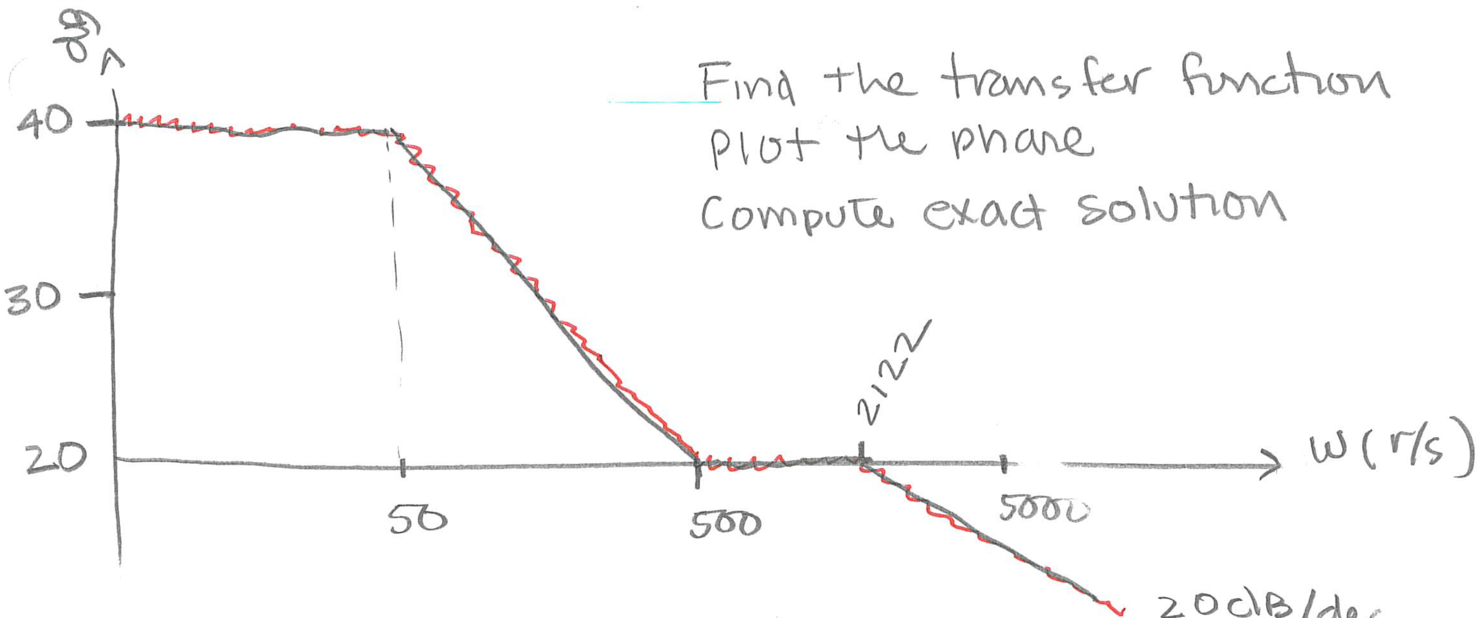


Figure 4 - Composite phase plot

$$H(s) = \frac{1}{8} \cdot \frac{j \cdot \omega}{\left(1 + \frac{j \cdot \omega}{4}\right) \cdot \left(1 + \frac{j \cdot \omega}{10}\right)^2}$$

Problem 14.24



Find the transfer function
Plot the phase
Compute exact solution

1) From DC to 50 r/s, Gain = 40 dB

So $20 \log_{10}(K) = 40$, $\log_{10}(K) = \frac{40}{20} = 2$

$K = 10^2 = 100$

2) At $\omega = 50$, function starts dropping at 20 dB/decade

∴ there is a pole at $\omega = 50 \longrightarrow \frac{1}{1 + j\omega/50}$

3) At $\omega = 500$, Response Flattens out.

∴ There is a zero at $\omega = 500 \longrightarrow \left(1 + j\frac{\omega}{500}\right)$

4) At $\omega = 2122$, the response begins falling at 20dB/dec

∴ there is a pole at 2122 $\longrightarrow \frac{1}{1 + j\frac{\omega}{2122}}$

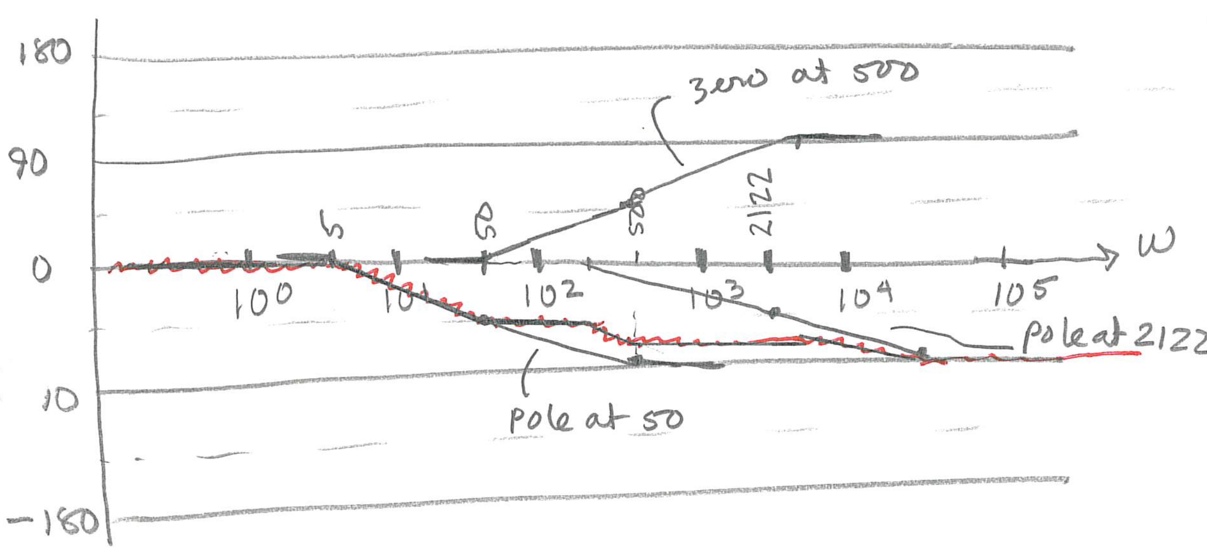
Multiply out the terms

Note ϕ at ∞ $= -90^\circ$

$$H(w) = 100 \times \frac{(1 + jw/500)}{(1 + jw/50)(1 + jw/2122)}$$

Now plot the phase.

Scale the plot. Freq from 0.1 x lowest pole or zero or 5 r/s to 10 times highest = 21K



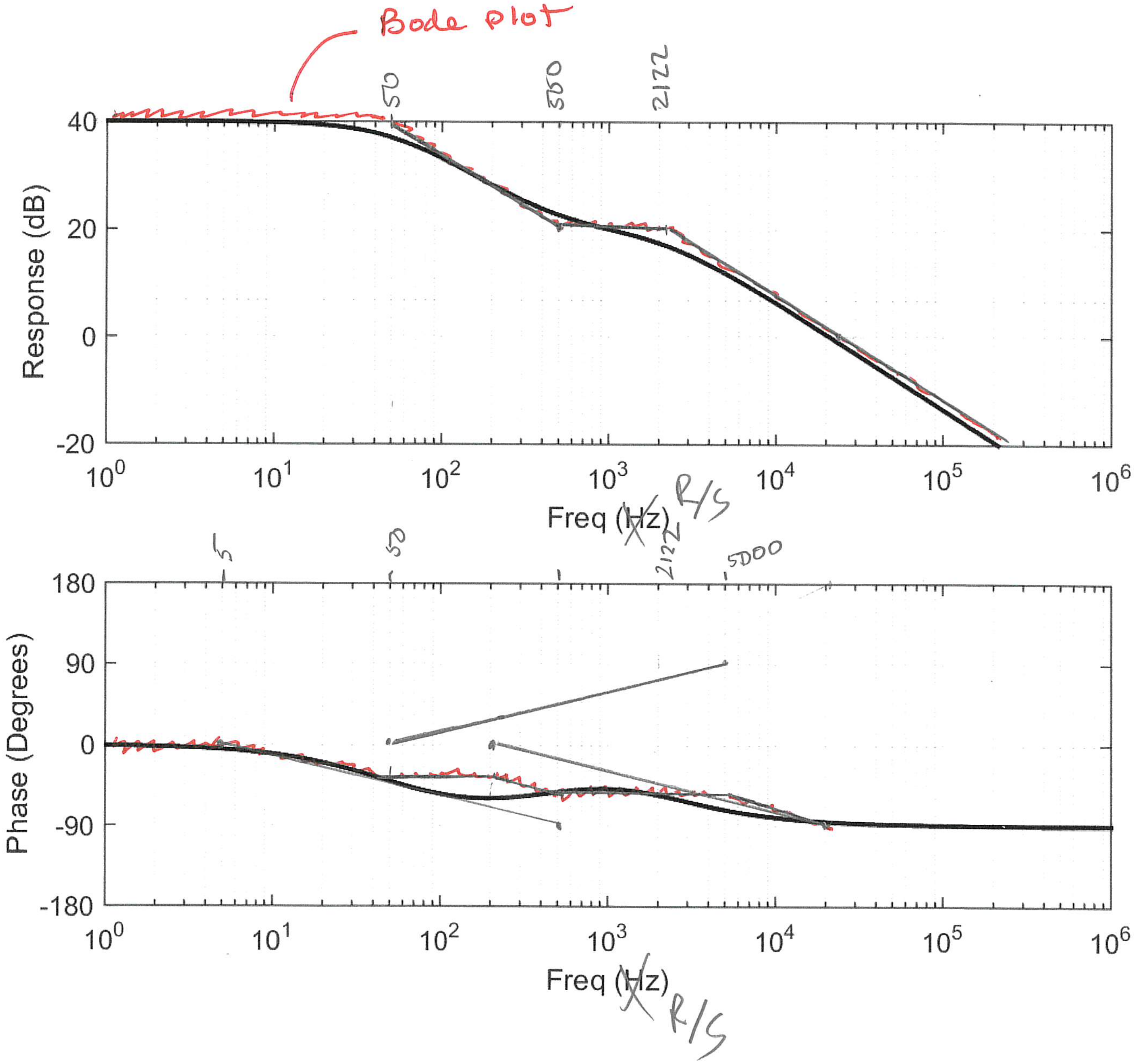
so 1 r/s to 100K

six decades

$\log_{10} 2122 = 3.32$

$\log_{10}(50) = 1.7$

L10


Bode plot
50
350
2122
40
20
0
-20
Response (dB)
10^0
10^1
10^2
10^3
10^4
10^5
10^6
Freq (~~Hz~~) R/S
5
50
2122
5000
180
90
0
-90
-180
Phase (Degrees)
10^0
10^1
10^2
10^3
10^4
10^5
10^6
Freq (~~Hz~~) R/S

```
% Filename:         ComputeFreqResp.m
% Author:           Barry Dorr
% Last Modified:    10-23-2012

% This script computes the frequency response for a s-domain transfer function.

clear all;
close all;
```

in hz

Start at 1 rad/sec

```
StartFreq = 1/2/pi;     % Lowest frequency to plot.
NumDec = 6;             % Number of decades to plot.
PtsPerDec = 200;        % Number of frequency points plotted per decade.

% Pre-allocate results matricies.
MagResp = zeros(NumDec*PtsPerDec,1);    % Matrix containing magnitude response.
PhaseResp = zeros(NumDec*PtsPerDec,1);  % Matrix containing phase response.
Freq = zeros(NumDec*PtsPerDec,1);       % Matrix containing plot frequencies.


% Compute frequency response at frequency points uniformly-spaced on a log plot.
for i=1 : NumDec*PtsPerDec
    Freq(i) = StartFreq*10^(i/PtsPerDec);   % Evaluate at this frequency.
    s = 1j * 2*pi*Freq(i);                  % Determine complex frequency.

    H = 100*(1+s/500)/((1+s/50)*(1+s/2122));    % Compute response at this frequency.

    MagResp(i) = abs(H);                    % Place magnitude response in result matrix.
    PhaseResp(i) = angle(H)*360/(2*pi);     % Place phase response in result matrix.
end


% Plot magnitude and phase.
subplot(2,1,1);
```

plot radians

```
HMag=semilogx(Freq*2*pi,20*log10(MagResp), 'color','k', 'linewidth', 2); % Plot magnitude
set(gca,'FontSize',12);
xlabel('Freq (Hz)');
ylabel('Response (dB)');
ylim([-20 40]);
grid on;
```

plot radians

```
subplot(2,1,2);
HPhs = semilogx(Freq*2*pi,PhaseResp, 'color','k', 'linewidth', 2); % Plot phase
set(gca,'FontSize',12);
xlabel('Freq (Hz)');
ylabel('Phase (Degrees)');
ylim([-180 180]);
set(gca,'YTick',[-180, -90, 0, 90, 180]);
grid on;
```

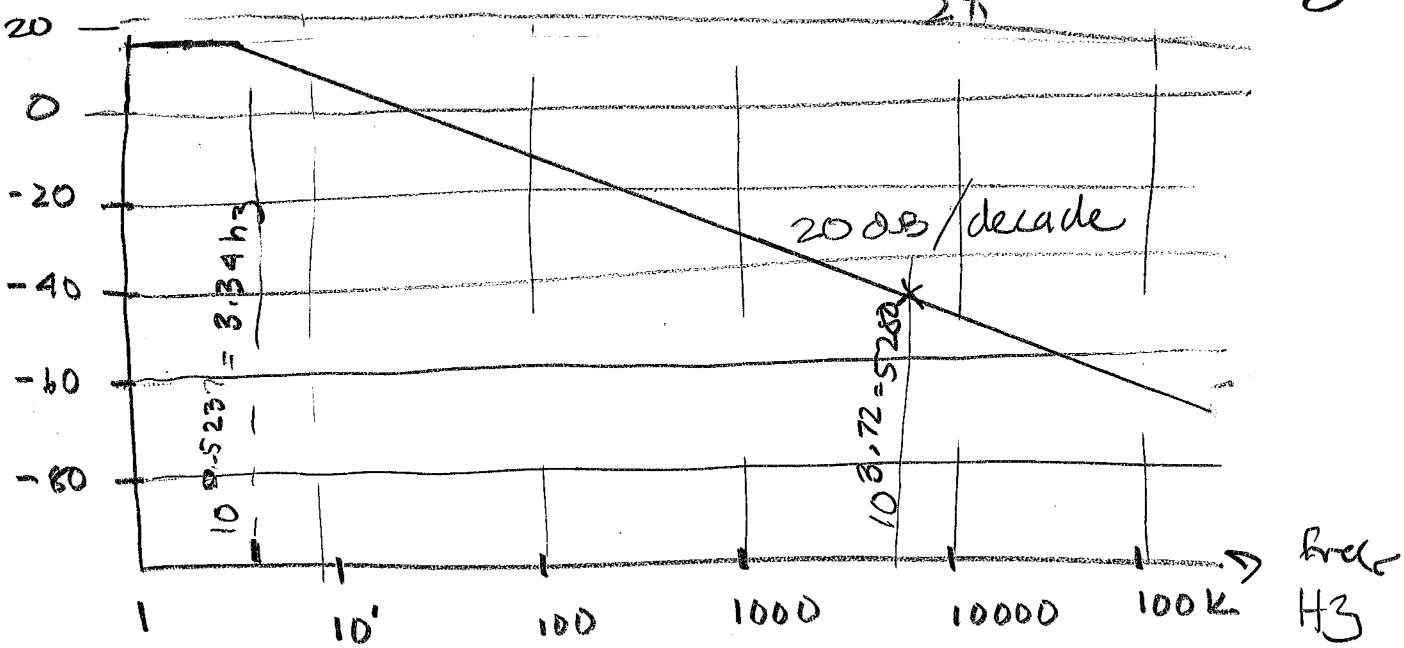
Example

Plot magnitude response as a function of frequency in Hz

$$H(j\omega) = 8 \times \frac{1}{1 + j\omega/21}$$

Also find magnitude at 5280 hz

$\omega_p = 21$ r/s so freq in hz $= \frac{21}{2\pi} = 3.34$ hz



1) $20\log_{10}(8) = 18.06$ dB → draw line

2) $\log_{10}(3.34) = 0.5237$ → mark pole freq

3) draw -20 dB/dec from constant line starting at pole

4) $\log_{10}(5280) = 3.72$ – Read about -47 dB

Now get better approximation

decades between pole freq and 5280 hz

$$10^x = \frac{5280}{3.34} \qquad x \cancel{\log_{10}(10)} = \log_{10}\left(\frac{5280}{3.34}\right)$$

$$x = \log_{10}\left(\frac{5280}{3.34}\right) = 3.20 \text{ decades}$$

$$\text{Resp at } 5280 = 18.06 + 3.20 \text{ decades} \times -20 \frac{dB}{dec}$$

$$= -45.91 \text{ dB}$$ Not bad!

What is the rolloff in dB/oct for a single pole?

$$\Delta dB = 20 \log_{10}\left(\frac{2\omega}{\omega}\right)$$

$$= 20 \log_{10}(2) = 6.02 \text{ dB/octave}$$

so $\underbrace{20 \text{ dB/decade} \equiv 6 \text{ dB/octave}}_{\text{interchangeable}}$

How many octaves between F_1, F_2 ?

$$2^n = \frac{f_2}{f_1} \qquad n \ln(2) = \ln(f_2/f_1)$$

$$n = \frac{\ln(f_2/f_1)}{\ln(2)}$$ ← Really useful

Revisit last problem

Octaves between 5280 and pole freq.?

$$\text{Octaves} = \frac{\ln(5280/3.34)}{\ln(2)} = 10.62$$ do this on your fingers

So 18.06 dB − 10.62 octaves × 6 dB/octave

= −45.9 dB, → Same answer!

Poles and Zeros

$$H(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The "poles" are the values of s where the denominator polynomial is zero. The transfer function becomes infinite.

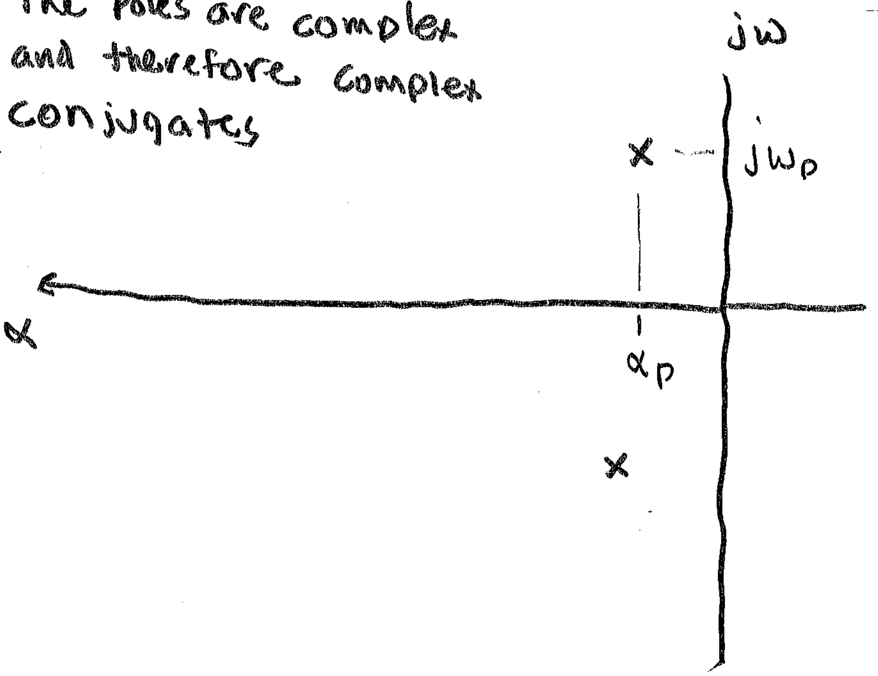
When there is also a polynomial in the numerator, the "zeros" are the values of s where the numerator evaluates to zero. The transfer function becomes zero.

So

$$H(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{1}{(s-p_1)(s-p_2)}$$

Pole values

The poles are complex and therefore complex conjugates



Poles are

$-\alpha_p \pm j\omega_p$

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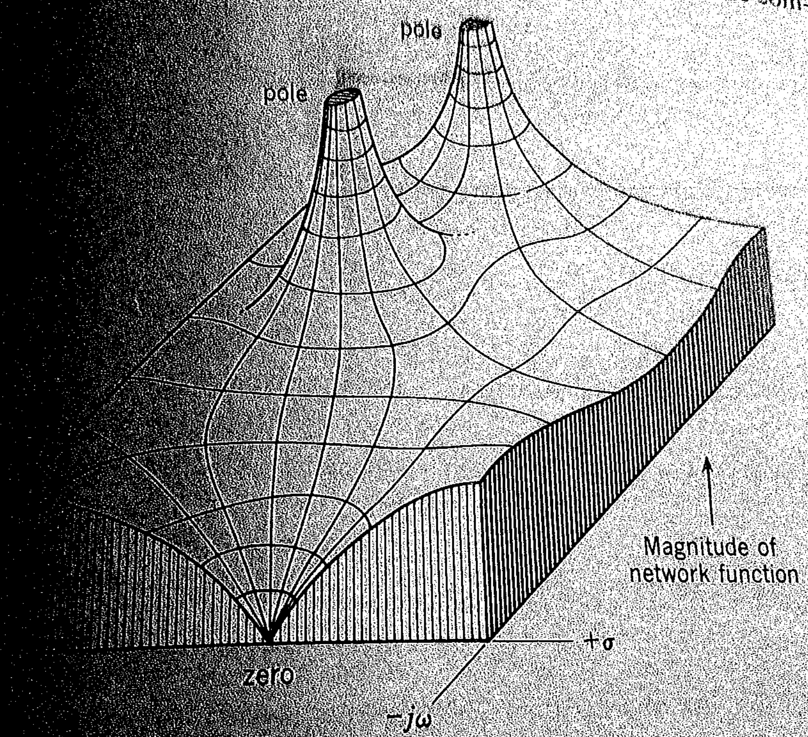


The magnitude of a network function shown as a func-
omplex frequency with two poles and one zero.

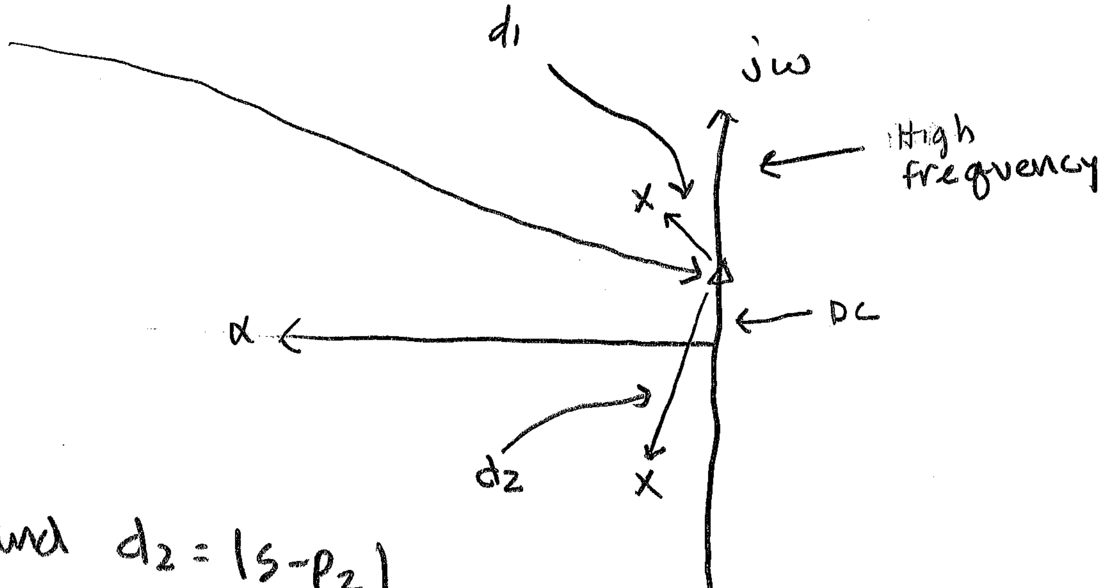
The magnitude of the transfer function is

$$|H(s)| = \frac{1}{|s-p_1| \times |s-p_2|}$$

when we measure frequency response, we use a sinewave input:

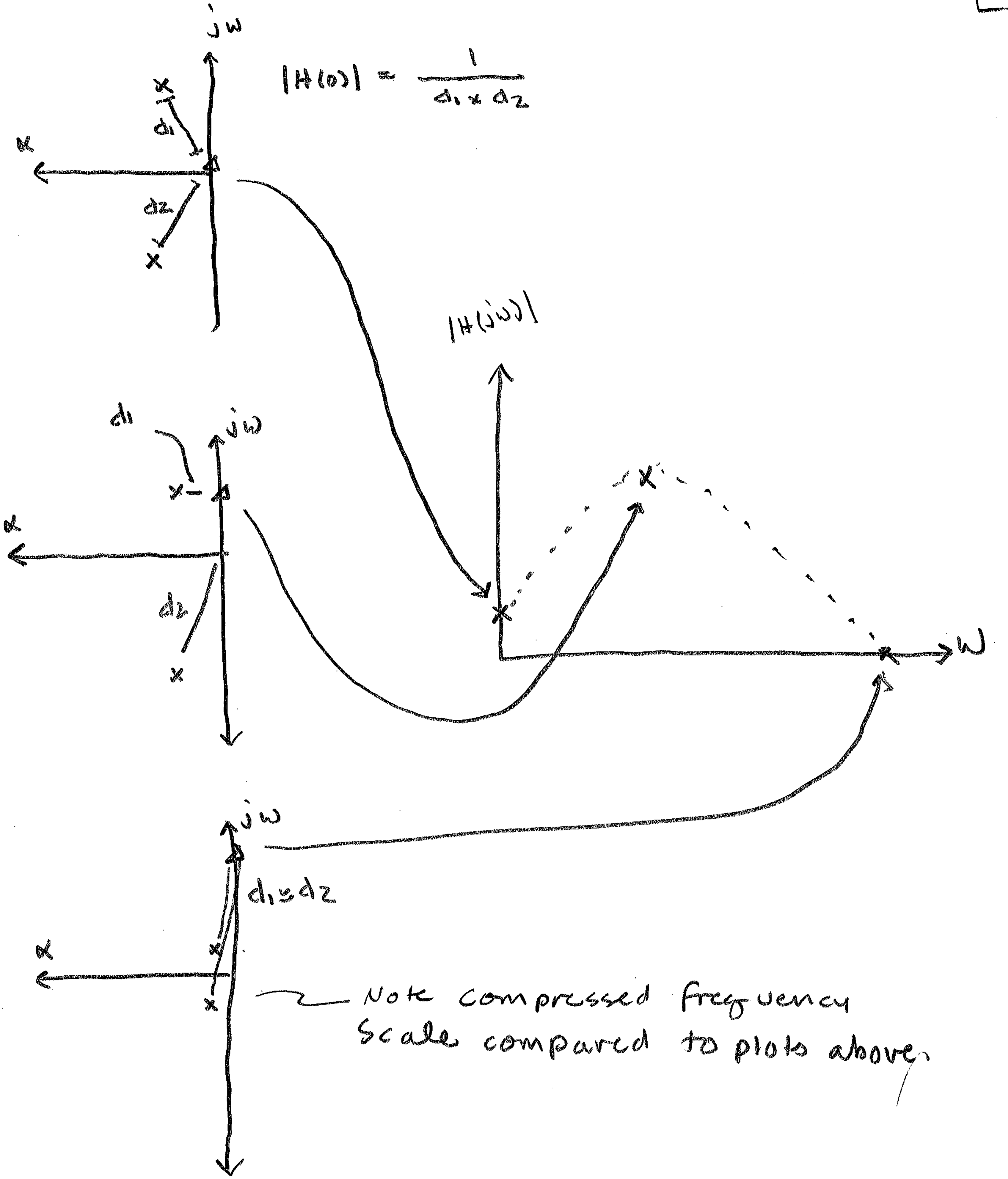
$$V_{in} = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

so $s = j\omega$



Note that

$d_1 = |s-p_1|$ and $d_2 = |s-p_2|$


jw
$|H(0)| = \frac{1}{d_1 \times d_2}$
d_1
d_2
α
$|H(jw)|$
jw
d_1
d_2
α
W
jw
$d_1 \approx d_2$
α
Note compressed frequency scale compared to plots above.

Now we know what poles and zeros are. How do they relate to Bode plots?

$$H(j\omega) = \frac{K}{\left(1 + \frac{j\omega}{\omega_{p1}}\right)\left(1 + \frac{j\omega}{\omega_{p2}}\right)}$$

We showed that the exact response at ω_p (for a single pole) is $-3\text{ dB} = \frac{1}{\sqrt{2}}$

For a single pole, $H(s) = \frac{1}{s - p_1}$

$$|H(s)| = \left|\frac{1}{s-p_1}\right| = \frac{1}{\sqrt{s^2 + p_1^2}}$$

So if $s = p_1$, then the response is $\frac{1}{\sqrt{2}}$ which is -3 dB.

So if we have a single pole at $\alpha = -3$ r/s, $\omega_p = 3$ r/s

Poles and zeros of Transfer functions

$$H(s) = \frac{s^2+5s+4}{s^2+4s+13} = \frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)}$$

$$= \frac{(s - -1)(s - -4)}{(s-(-2+j3))(s-(-2-j3))}$$

Zero frequencies are the frequencies where the transfer function is zero.

Pole frequencies are the frequencies where the transfer function is infinite.

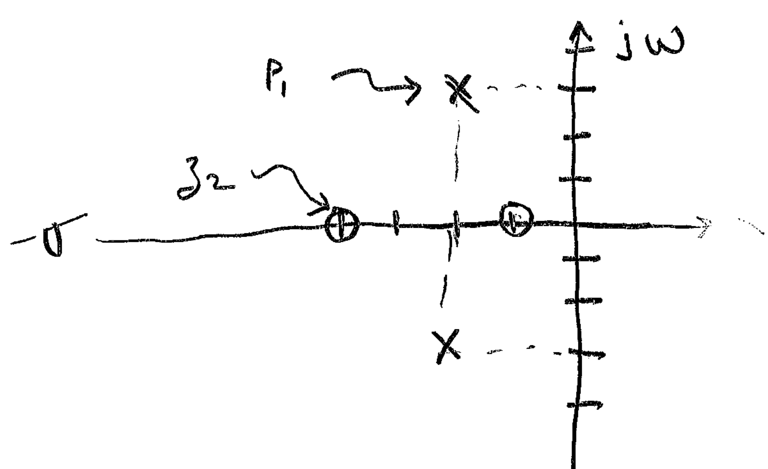
so

$$H(s) = \frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)}$$

For the function above,

$z_1 = -1, \quad z_2 = -4$

$p_1 = (-2+j3), \quad p_2 = (-2-j3)$



Pole-zero plot in the s-domain

We can get the frequency response from the pole-zero plot.

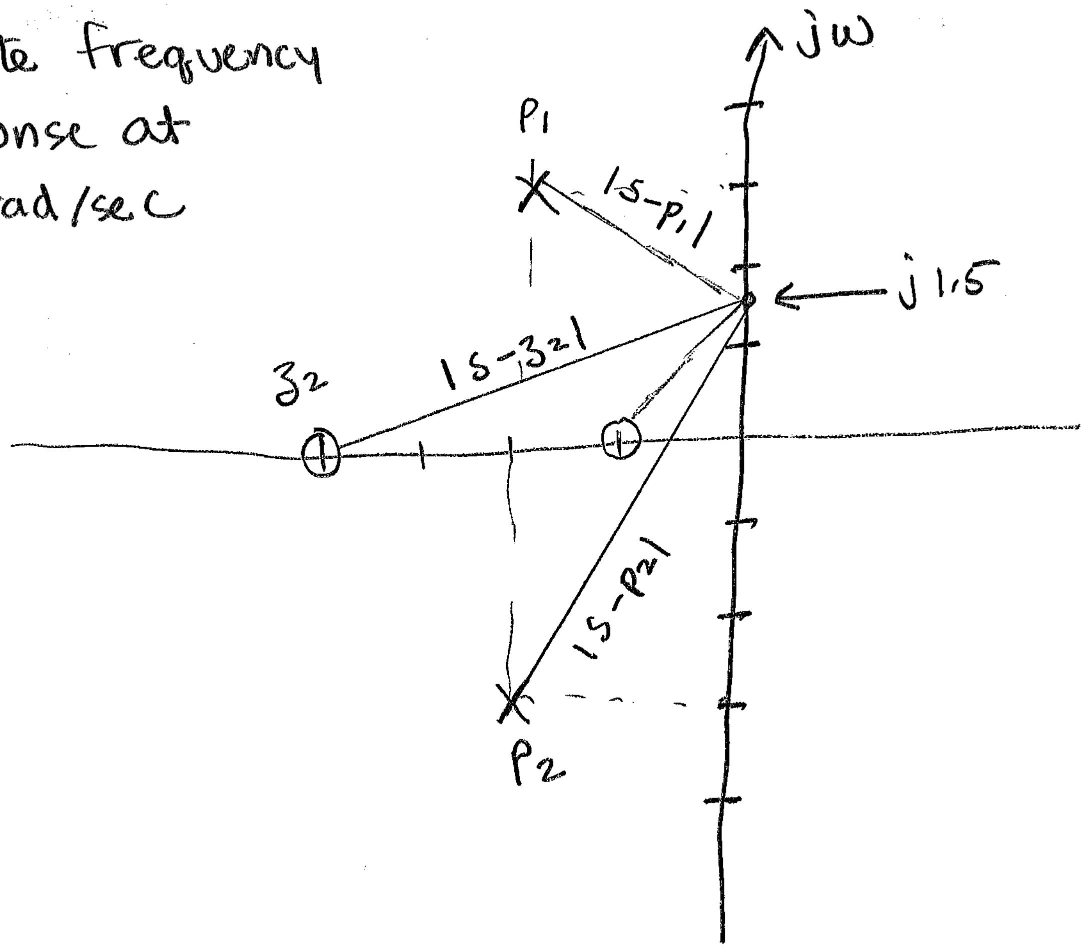
$$H(s) = \frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)}$$

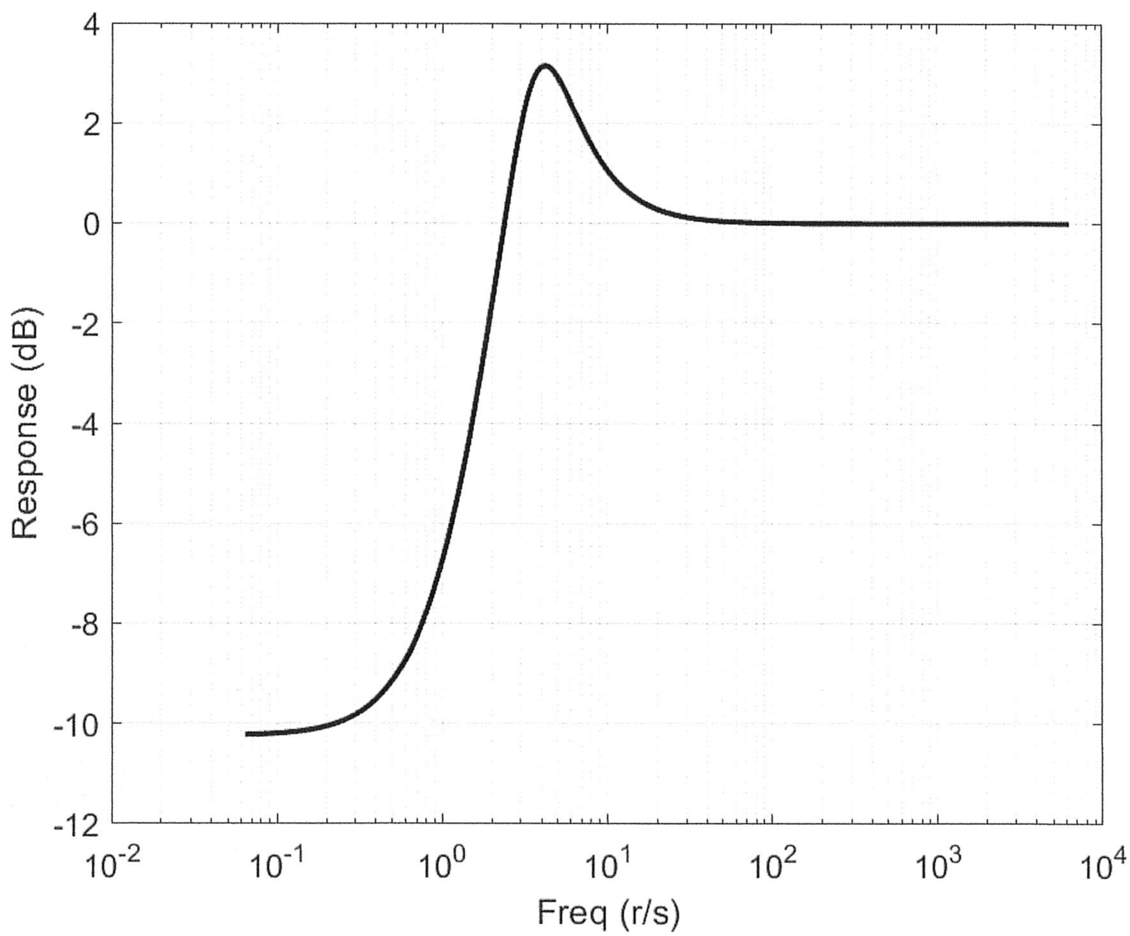
To compute frequency response, we let $s = j\omega$

$$|H(s)| = \frac{|s-z_1| \cdot |s-z_2|}{|s-p_1| \cdot |s-p_2|}$$

Each term is the distance between the pole or zero and $j\omega$.

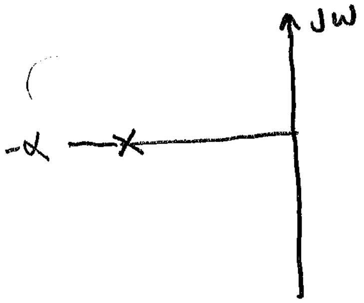
compute frequency response at 1.5 rad/sec



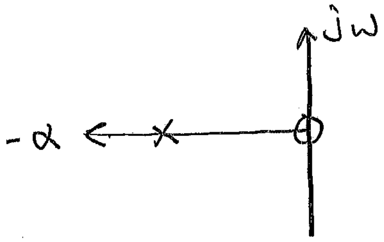


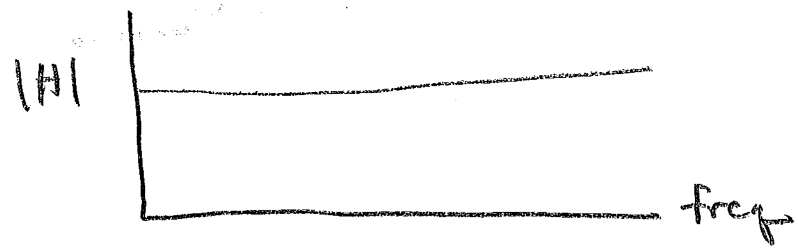
$$H(s) = \frac{s^2+5s+4}{s^2+4s+13}$$

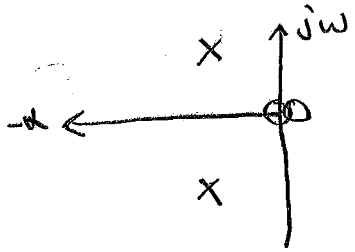
MATCH 'EM UP!


jw
-α


0 dB
0 dB
|H|
freq


jw
-α


|H|
freq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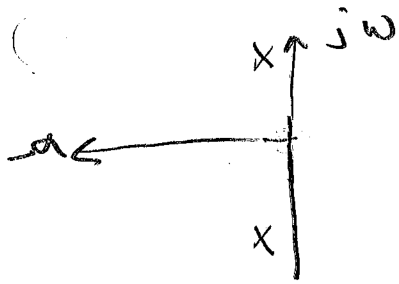
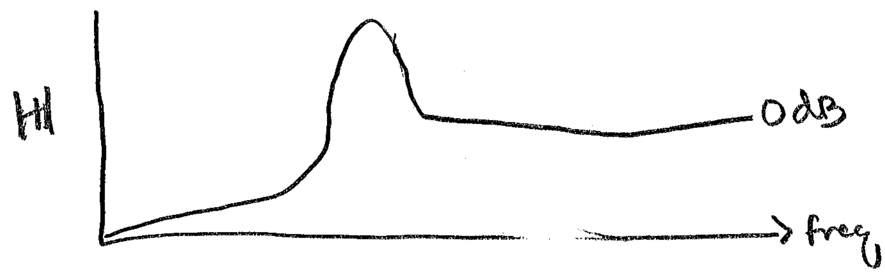
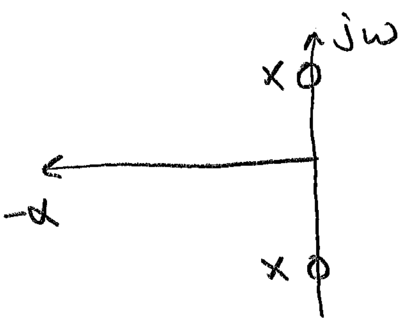
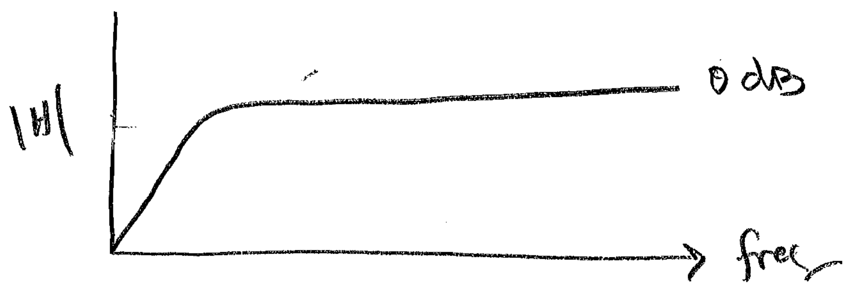
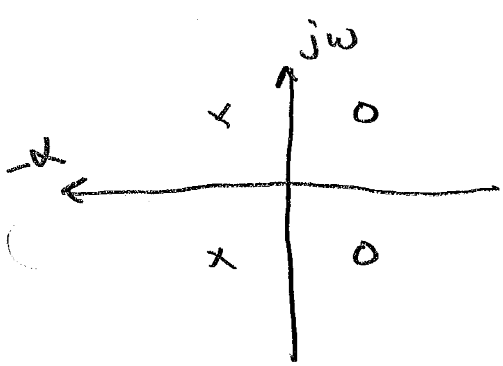

jw
-α


|H|
0 dB
freq


jw
-α


|H|
0 dB
freq


jw
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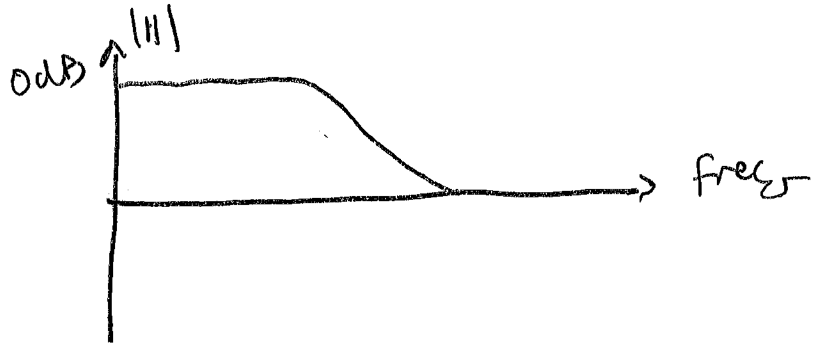
0 dB
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