LECTURE 17 BODE PLOTS I

LAST LECTURE — Computed frequency response of a simple circuit.

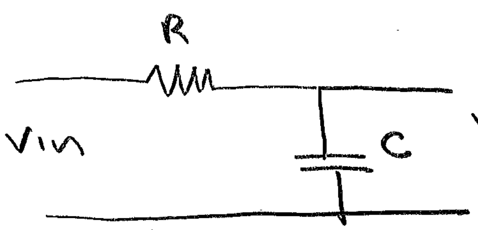
— Computed seperate plots for magnitude and phase.

— Plotted in Matlab.

— Asymptotic Analysis

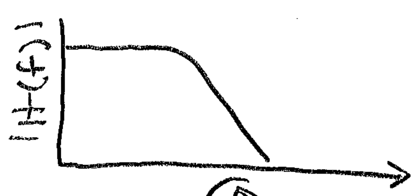
Today's lecture: Bode plots – a quick way to approximate the frequency response of a circuit.

Bode plot – helps you get an intuitive feel for circuits



$$\frac{V_o(s)}{V_{in}(s)} = H(s) = \frac{\frac{1}{sC}}{R + 1/sC} = \frac{1}{1 + sCR} = \frac{1}{1 + \frac{j\omega}{1/RC}}$$

(A)



(B)

(C)

$$\frac{V_o(s)}{V_{in}(s)} = H(s) = H(j\omega) = \frac{1}{1 + \frac{j\omega}{1/RC}} = \frac{1}{1 + \frac{j\omega}{\omega_p}}$$

$$\frac{1}{RC} = \omega_p$$

Exact frequency response

$$H_{dB}(\omega) = 20\log_{10}\left|\frac{1}{1 + \frac{j\omega}{\omega_p}}\right| = 20\log_{10}\left(\frac{1}{\sqrt{1 + \frac{\omega^2}{\omega_p^2}}}\right)$$

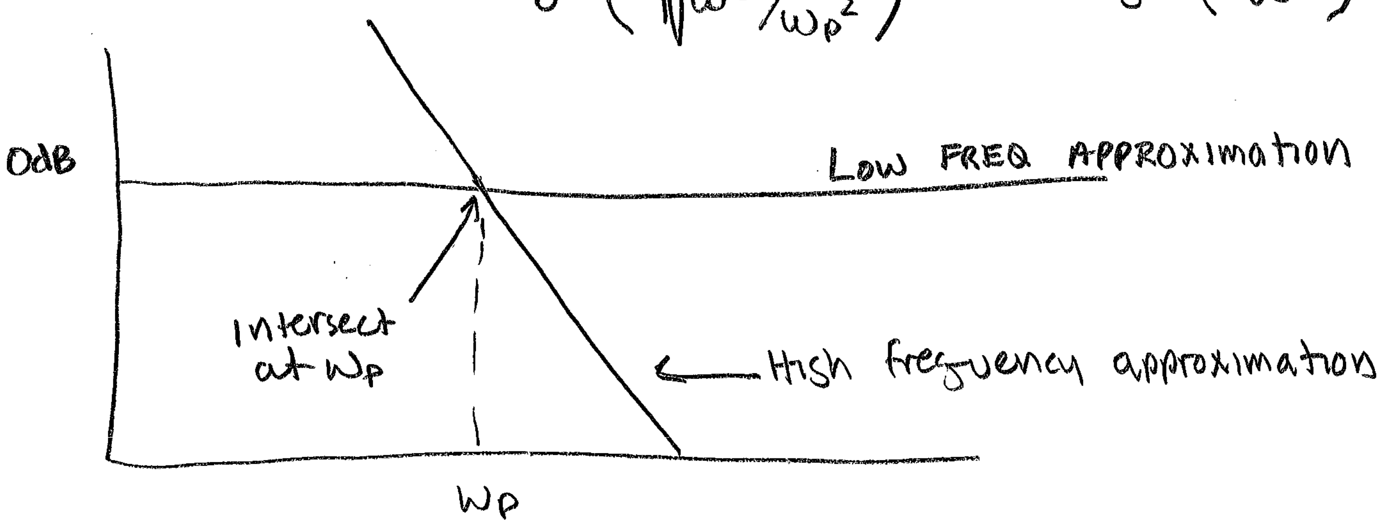
Approximate frequency response

At low frequencies

$$H_{dB} \cong 20\log_{10}\left(\frac{1}{\sqrt{1+0}}\right) = 0\text{ dB}$$

At high frequencies

$$H_{dB}(\omega) \cong 20\log\left(\frac{1}{\sqrt{\omega^2/\omega_p^2}}\right) = 20\log\left(\frac{\omega_p}{\omega}\right)$$

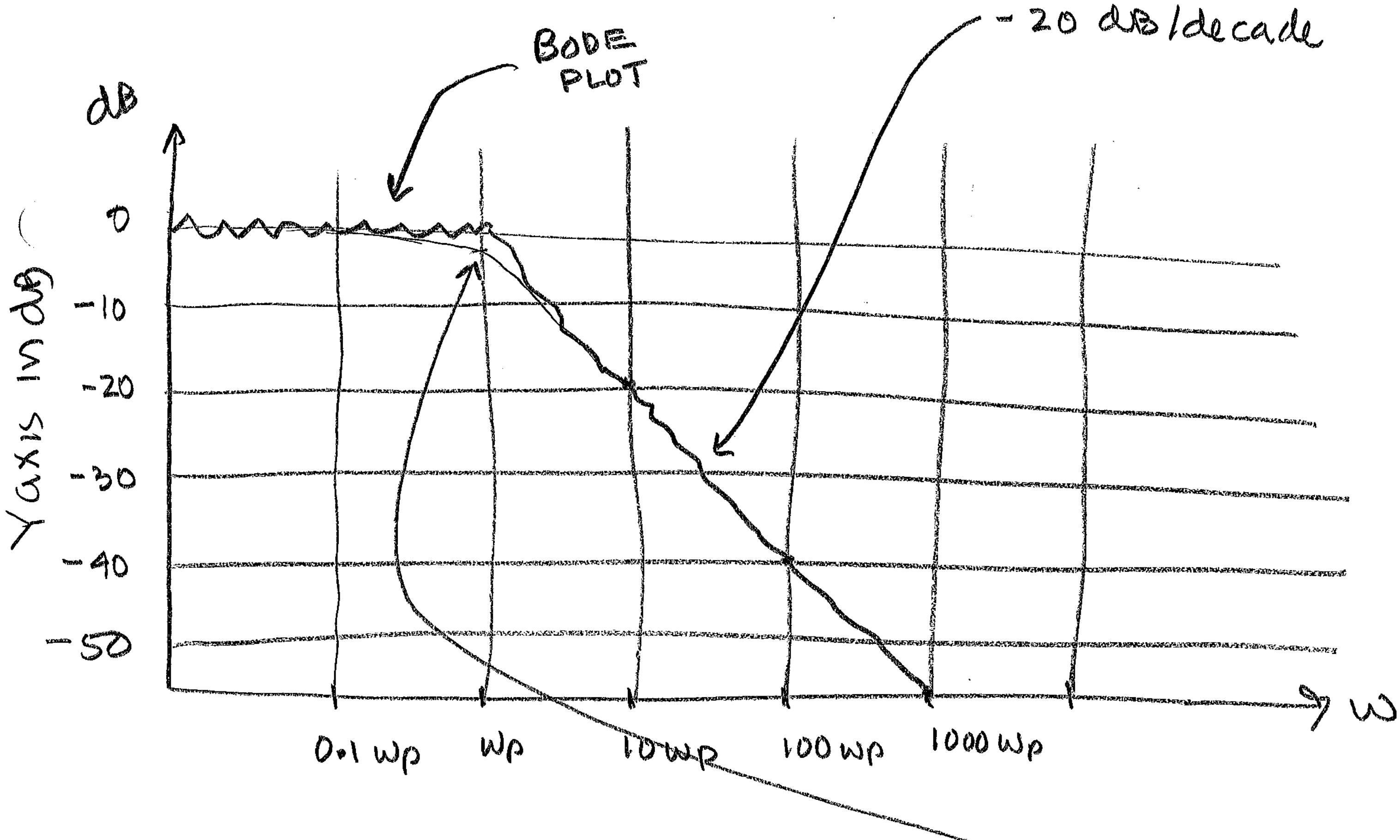


Look Further at high frequency approximation

ω	$20 \log_{10} (\omega_p/\omega)$
ω_p	0 dB
$10\,\omega_p$	−20 dB
$100\,\omega_p$	−40 dB

0 dB at ω_p

slope is −20 dB/decade



$$20 \log_{10} \left(\frac{1}{\sqrt{1 + \frac{\omega_p^2}{\omega_p^2}}} \right) = 20 \log \left(\frac{1}{\sqrt{2}} \right) = -3\,\text{dB}$$

constant term

zero at origin

zeros

Say

$$H(j\omega) = \frac{K\,(j\omega)\left(1+\frac{j\omega}{\omega_{z1}}\right)\left(1+j\omega/\omega_{z2}\right)}{\underbrace{\left(1+\frac{j\omega}{\omega_{P1}}\right)\left(1+j\omega/\omega_{P2}\right)}_{\text{Poles}}}$$

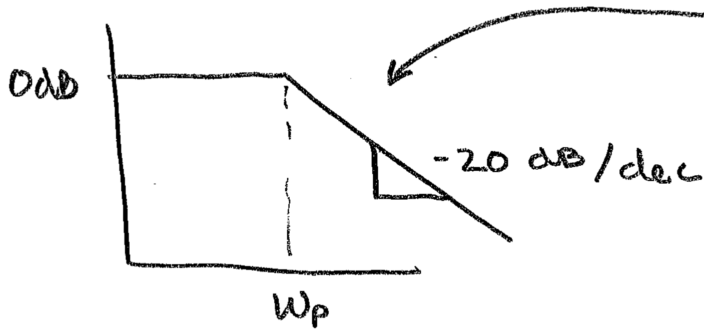
Frequency response

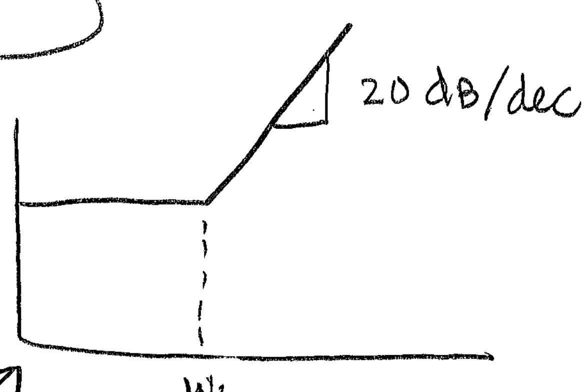
$$H_{dB}(j\omega) = 20\log_{10}\left[\frac{|T_1(j\omega)|\cdot|T_2(j\omega)|\cdots}{|T_3(j\omega)|\cdot|T_4(j\omega)|\cdots}\right]$$

$$H_{dB}(j\omega) = 20\log_{10}(|T_1(j\omega)|) + 20\log(|T_2(j\omega)|) - 20\log_{10}(|T_3(j\omega)|) - 20\log(|T_4(j\omega)|)\cdots$$

Shows we can handle terms independently and then add them.

We already saw pole behavior...





zero behavior

Transfer Function had constant, K

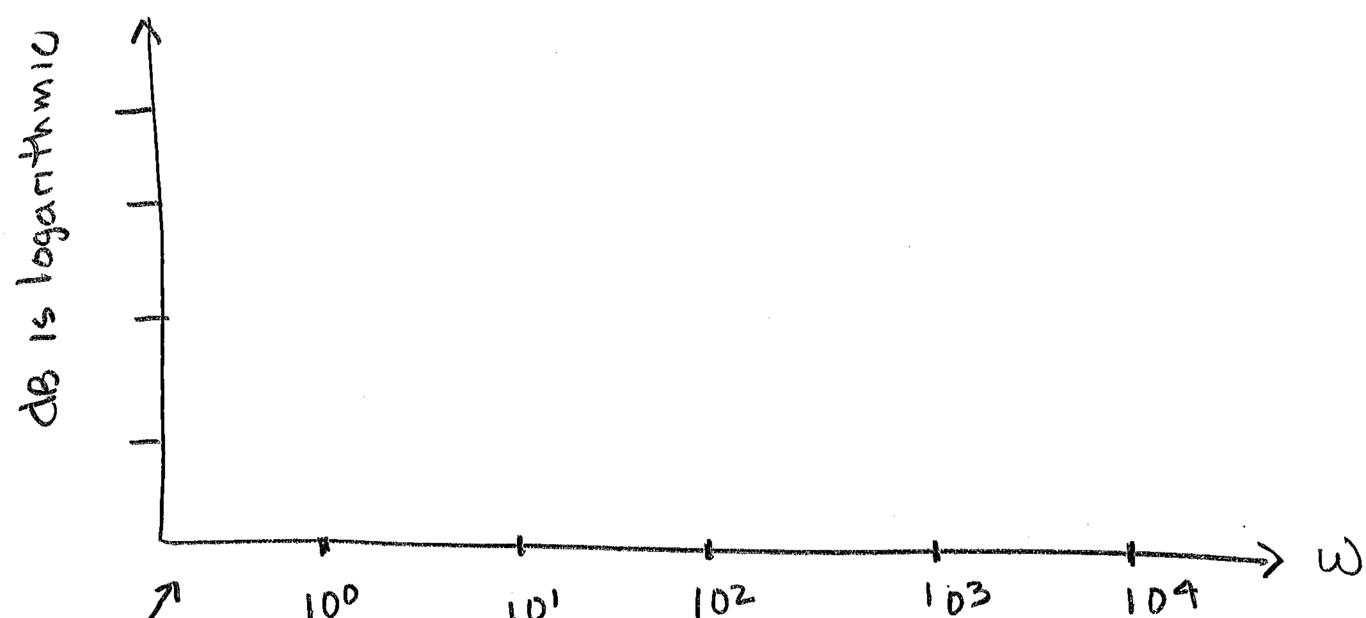
$20 \log_{10}(K) = $ flat line!

Transfer function had zero at origin

$20 \log_{10}(|j\omega|)$

ω	$20 \log(\lvert j\omega \rvert)$
1	0 dB
10	20 dB
100	40 dB

line segment has slope of 20 dB/decade. Goes through 0 dB at $\omega = 1$ r/s

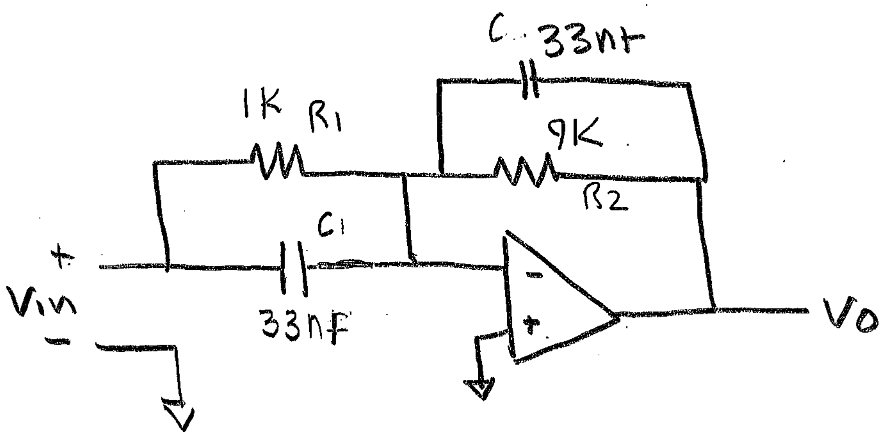


Logarithmic axis

Where is DC?

Example:

Sketch a Bode plot for the circuit below



$$\frac{V_o}{V_{in}} = -\frac{R_2 // Z_C}{R_1 // Z_C} \qquad R \| C = \frac{R \times 1/sC}{R + 1/sC} = \frac{R}{1+sCR}$$

$$\text{so}\quad H(s) = -\frac{\frac{R_2}{1+sCR_2}}{\frac{R_1}{1+sCR_1}} = \underbrace{-\frac{R_2}{R_1}}_{\text{Constant term}} \frac{\left(1+\frac{s}{1/R_1C}\right) \leftarrow \text{Zero}}{\left(1+\frac{s}{1/R_2C}\right) \leftarrow \text{Pole}}$$

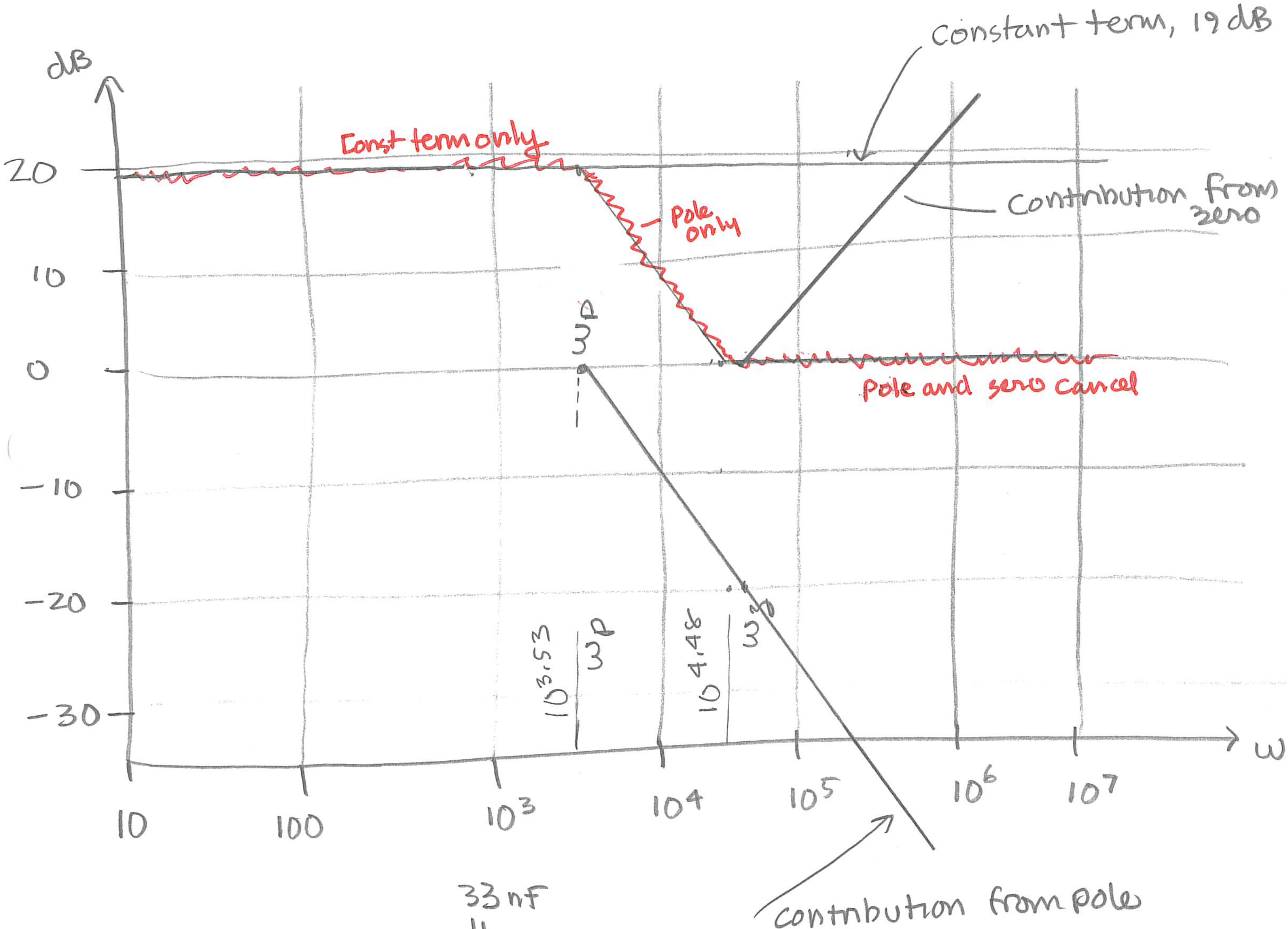
$$\frac{R_2}{R_1} = 9, \quad 1/R_1C \overset{= \omega_z}{=} 30.3E3 \text{ r/s} = 10^{4.48} \qquad \frac{1}{R_2C} \overset{= \omega_p}{=} 3.37E3 \text{ r/s} = 10^{3.53}$$

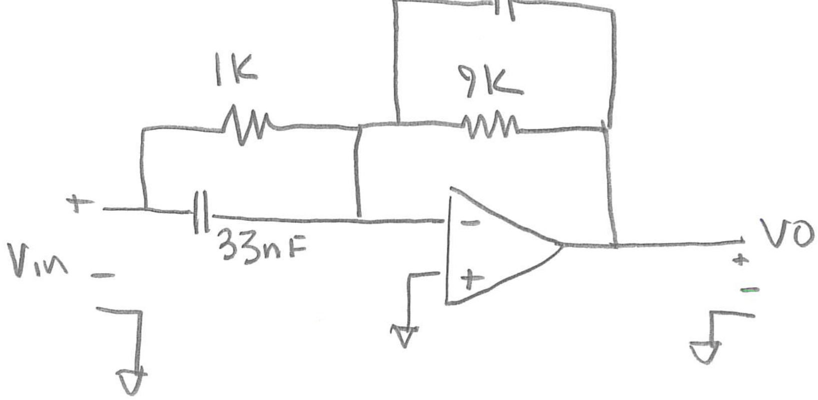
$$\text{so}\quad H(j\omega) = -9 \times \frac{1 + s/30.3E3}{1 + \frac{s}{3.37e3}}$$

Constant term
$20 \log_{10}(9) = 19.08\,dB$

Zero - Flat line, 20 dB/decade above ω_z

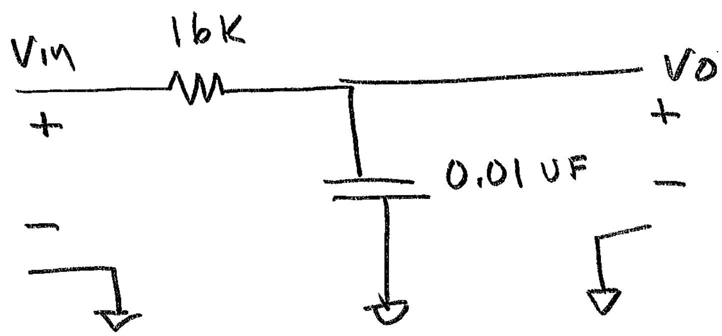
Pole - Flat line, -20 dB/decade above ω_p





Check with Asymptotic analysis

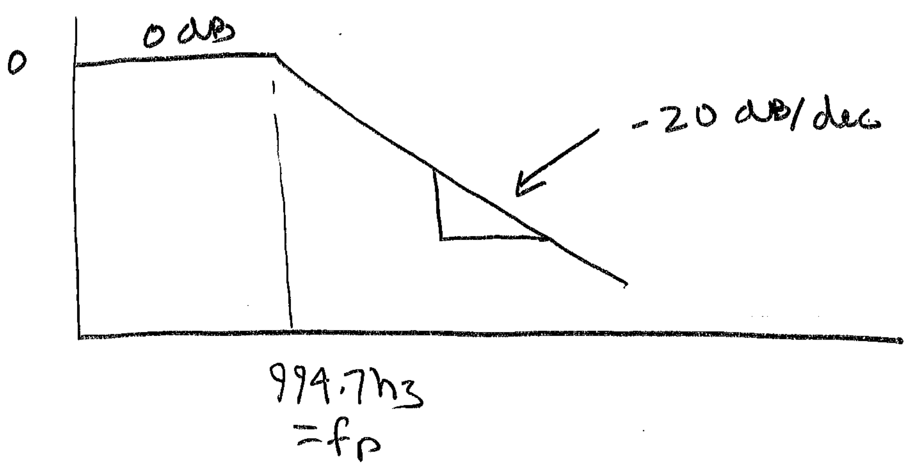
Reality check... Is this practical?



Find $H_{dB}(153\,\text{khz})$

$$H(s) = \frac{1/sC}{R + 1/sC} = \frac{1}{1+sCR} = \frac{1}{1+\frac{j\omega}{1/RC}} = \frac{1}{1+\frac{j\omega}{\omega_p}}$$

so $\omega_p = \frac{1}{RC}$ or $f_{pole} = \frac{1}{2\pi RC} = \underline{994.7\text{ hz}}$



How many decades between 153 khz and 995 hz?

$10^{dec} = \frac{153K}{995}$ or $dec \times \underbrace{\cancel{\log_{10}(10)}}_{=1} = \log_{10}\left(\frac{153K}{995}\right)$

$dec = \log_{10}\left(\frac{153K}{995}\right) = 2.19\text{ dec}$ so $0 - 20\frac{dB}{dec} \times 2.19\text{ dec}$

$= -43.8\text{ dB}$

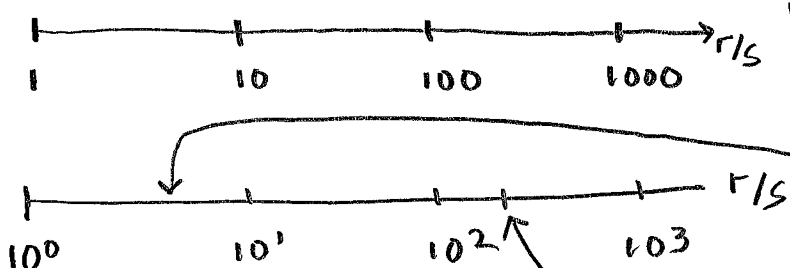
PRACTICE PROBLEM 14.4, Pg 626

Sketch Bode plot for

$$H(j\omega) = \frac{50 j\omega}{(j\omega+4)(j\omega+10)^2}$$

Put in form used on sheet 4

$$H(j\omega) = \underbrace{\frac{50}{4\times10^2}}_{\text{constant term}} \times \frac{\overset{\text{zero at origin}}{j\omega}}{\underbrace{\left(1+\frac{j\omega}{4}\right)}_{\text{single pole at } \omega=4}\underbrace{\left(1+\frac{j\omega}{10}\right)^2}_{\text{double pole at } \omega=10}}$$



where is 4 rad/sec?

$\log_{10}(4) = 0.602$

Where is 31 Hz??

$\log_{10}(31\times2\pi) = 2.28$

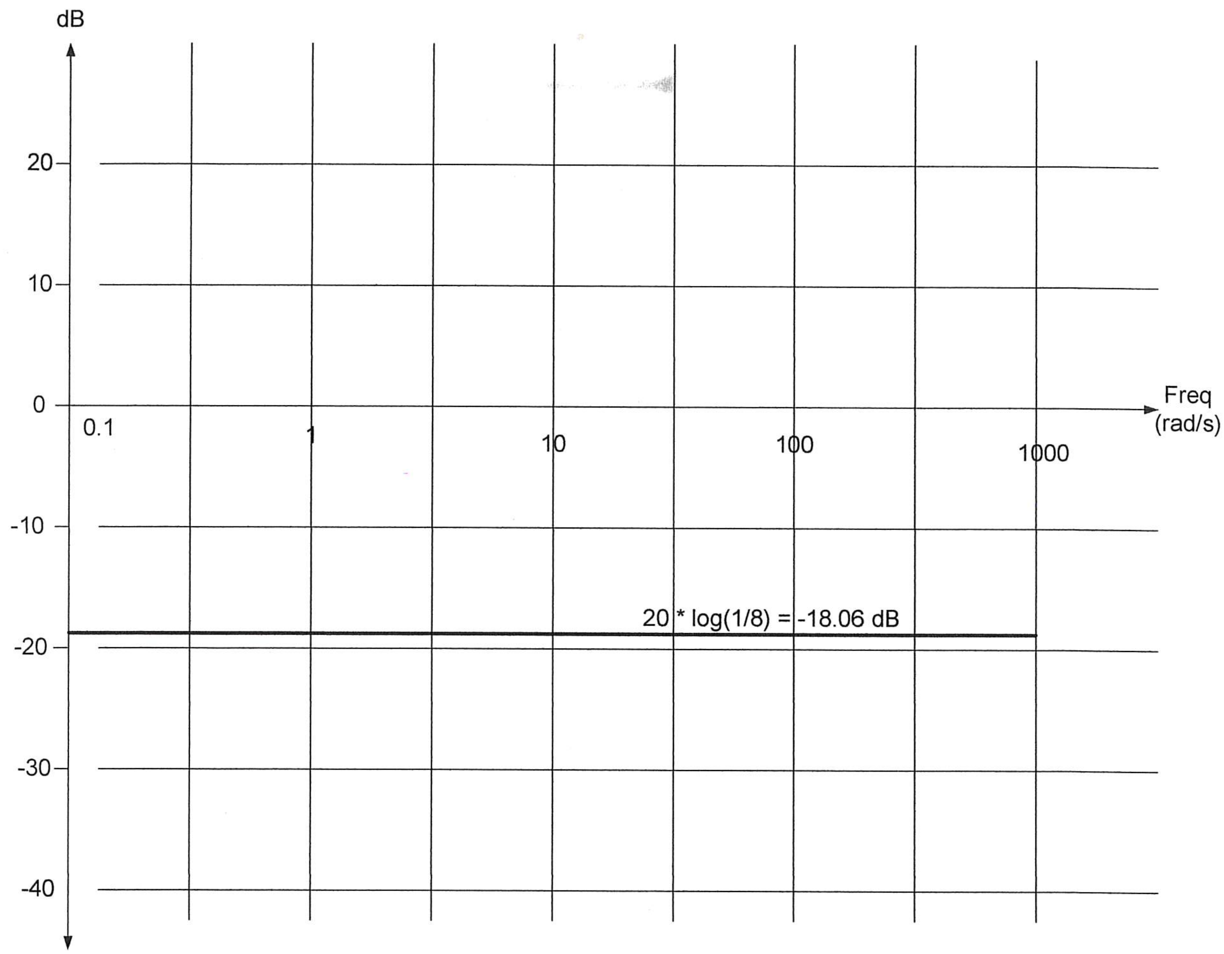


Figure 1 - Plot the constant Term

$$H(s) = \frac{1}{8} \cdot \frac{j \cdot \omega}{\left(1 + \frac{j \cdot \omega}{4}\right) \cdot \left(1 + \frac{j \cdot \omega}{10}\right)^2}$$

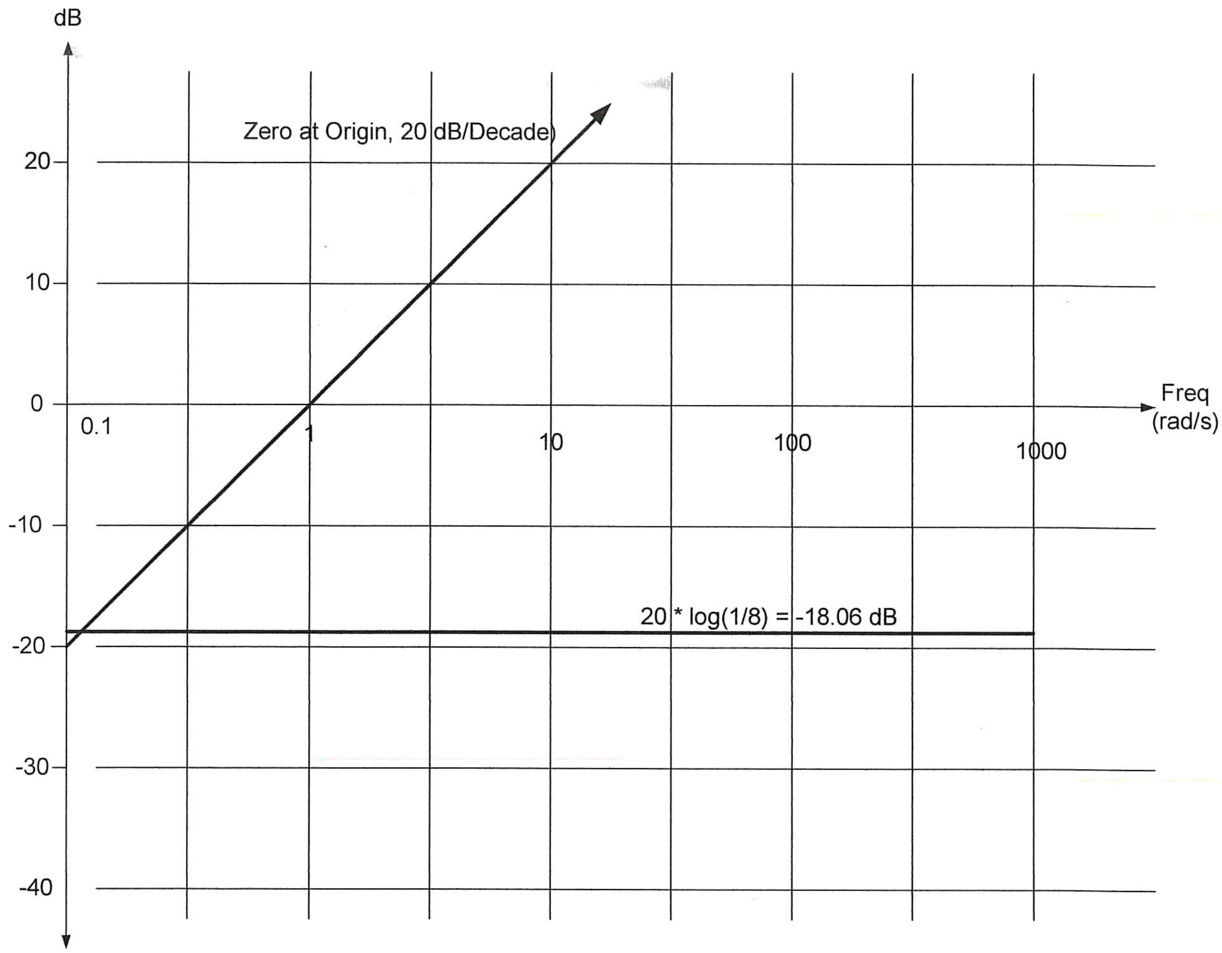


Figure 2 - Add Zero at origin

$$H(s) = \frac{1}{8} \cdot \frac{j \cdot \omega}{\left(1 + \frac{j \cdot \omega}{4}\right) \cdot \left(1 + \frac{j \cdot \omega}{10}\right)^2}$$

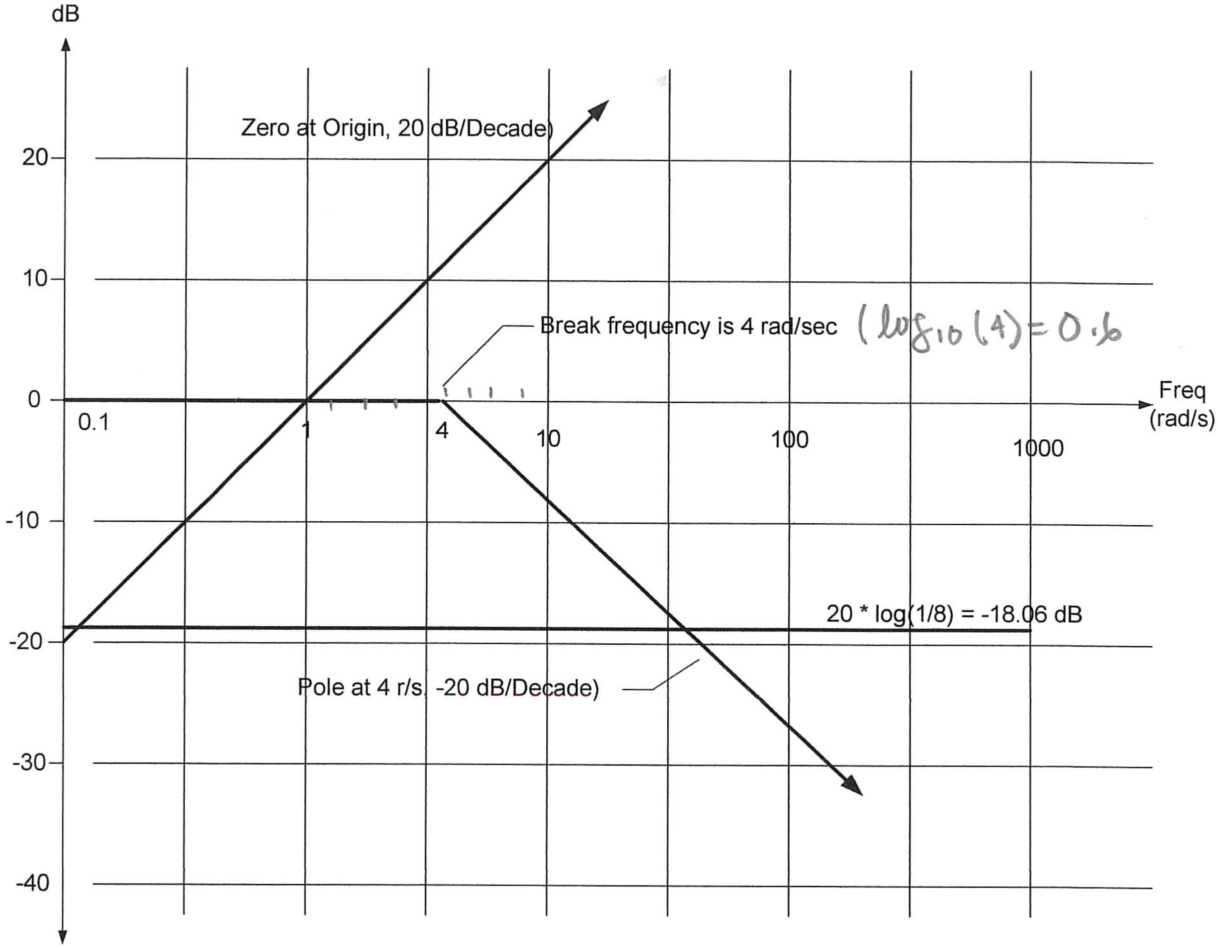


Figure 3 - Add the pole at 4 rad/s

$$H(s) = \frac{1}{8} \cdot \frac{j \cdot \omega}{\left(1 + \frac{j \cdot \omega}{4}\right) \cdot \left(1 + \frac{j \cdot \omega}{10}\right)^2}$$

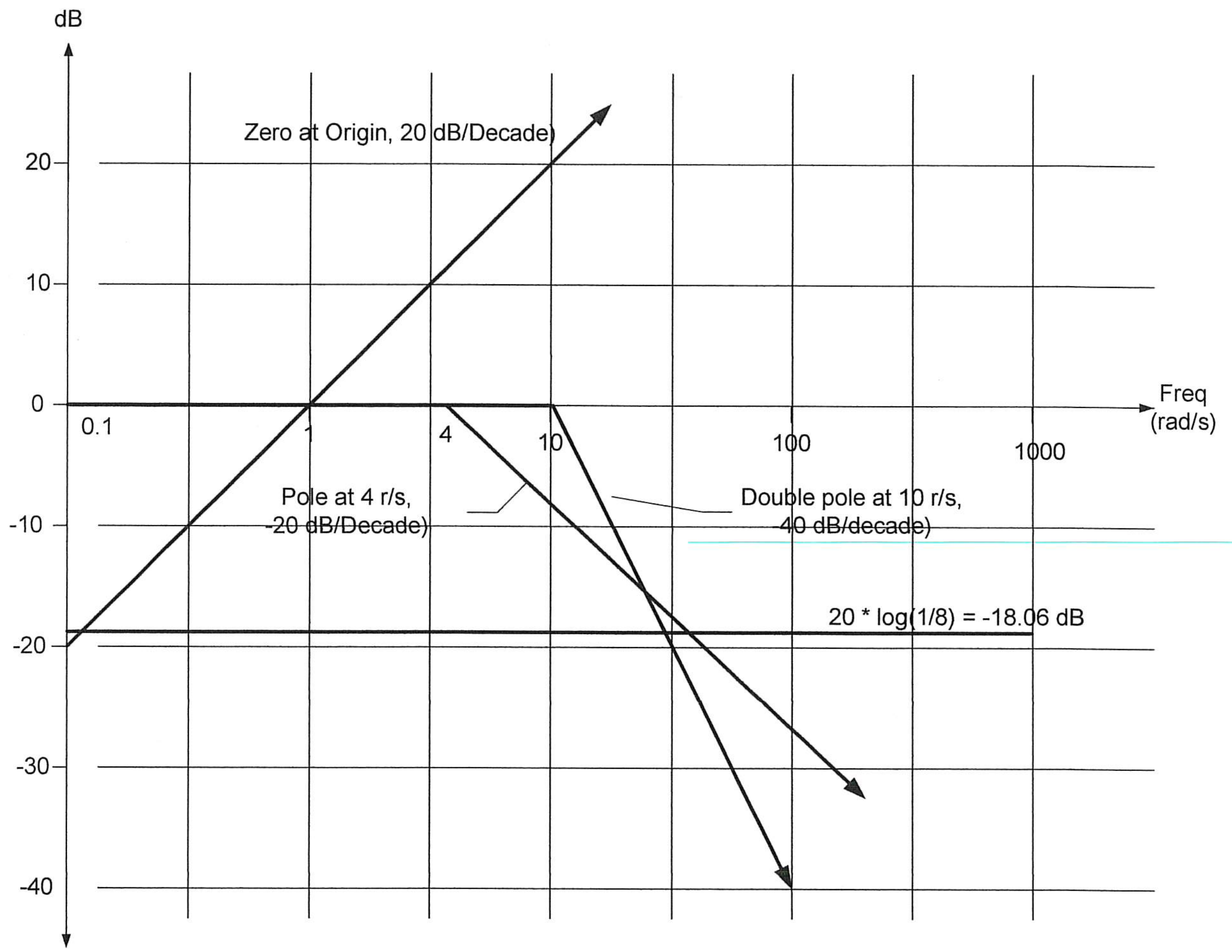


Figure 4 - Add the double pole at 10 r/s

$$H(s) = \frac{1}{8} \cdot \frac{j \cdot \omega}{\left(1 + \frac{j \cdot \omega}{4}\right) \cdot \left(1 + \frac{j \cdot \omega}{10}\right)^2}$$

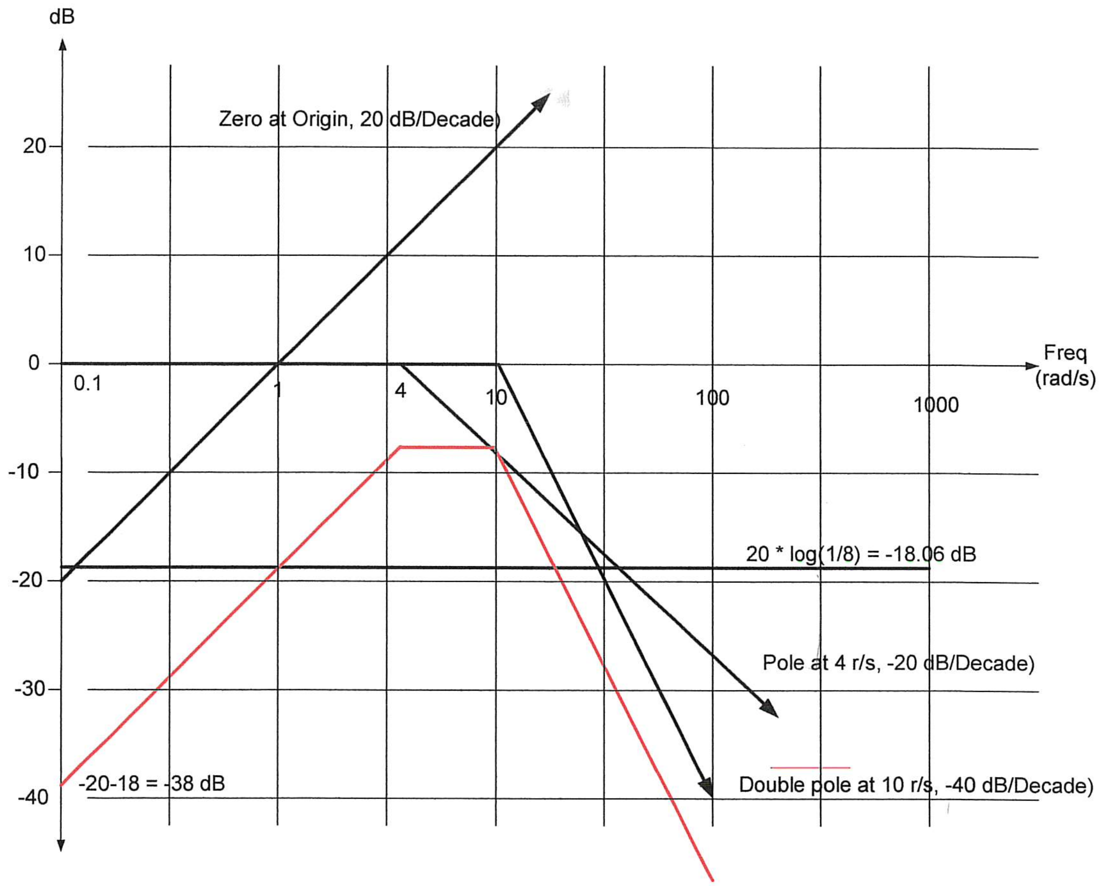


Figure 5 - Sum all the parts to Get magnitude

$$H(s) = \frac{1}{8} \cdot \frac{j \cdot \omega}{\left(1 + \frac{j \cdot \omega}{4}\right) \cdot \left(1 + \frac{j \cdot \omega}{10}\right)^2}$$

Quadratic Terms (Poles or zeros)

$as^2 + bs + C \longleftarrow$ ... in an expression

$s^2 + 2\zeta\omega_n s + \omega_n^2 \longleftarrow$ ... control systems and circuits. Most common way you'll see it.

$\omega_n^2\left[\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1\right] \longleftarrow$ manipulate a bit ...

$\omega_n^2\left[1 + 2\zeta \frac{j\omega}{\omega_n} + \frac{(j\omega)^2}{\omega_n^2}\right] \longleftarrow$ standard form

See EQ 14.15

If $\omega \ll \omega_n$, Magnitude $= 1 \rightarrow (20 \log_{10}(1) = 0\,dB)$

If $\omega \gg \omega_n$, magnitude $= \frac{\omega^2}{\omega_n^2}$

and

$$20 \log_{10}\left(\frac{\omega^2}{\omega_n^2}\right) = 40 \log_{10}(\omega/\omega_n)\ dB$$

and we see that slope is 40 dB/decade.

At ω_n, see Figure 14.2 pg 622 for frequency peaking.

... the "boingier" it is, the more frequency peaking we see!

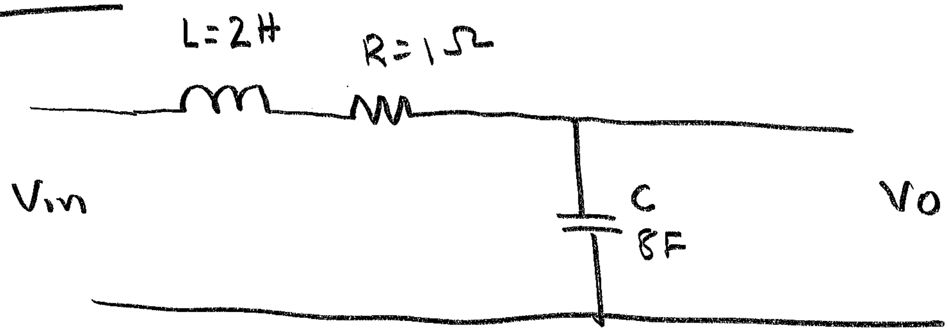
We started with $s^2 + 2\zeta \omega_n s + \omega_n^2$

and equated it to

$$\omega_n^2 \left[1 + 2\zeta \frac{j\omega}{\omega_n} + \frac{(j\omega)^2}{\omega_n^2} \right]$$

constant term

EXAMPLE



$$\frac{V_o}{V_{in}}(s) = H(s) = \frac{1/sC}{1/sC + R + sL}$$

$$= \frac{1}{LC} \times \frac{1}{s^2 + s \cdot \frac{R}{L} + \frac{1}{LC}}$$

← asymptotic analysis

DIGRESS TO CH 8

1) This is EQ 8.8 pg 320 for series RLC characteristic EQ

2) $\alpha = \frac{R}{2L} = 1/4$, $\omega_0 = \frac{1}{\sqrt{LC}} = 1/4$ → Critical damping

→

Now put into standard form for plotting

... that's just algebra

$$\overbrace{\frac{1}{LC}\cdot\frac{1}{s^2+s\frac{R}{L}+\frac{1}{LC}}}^{\text{CIRCUIT}} = \overbrace{K \times \frac{1}{1+2\zeta\frac{j\omega}{\omega_n}+\frac{(j\omega)^2}{\omega_n^2}}}^{\text{EQ 14.15}}$$

MULTIPLY

$$\frac{1}{1+sRC+s^2LC} = K \times \frac{1}{1+2\zeta\frac{j\omega}{\omega_n}+\frac{(j\omega)^2}{\omega_n^2}}$$

$K = 1$

$$\frac{1}{1+\underbrace{j\omega RC}+\frac{(j\omega)^2}{1/LC}} = \frac{1}{1+2\zeta\underbrace{\frac{j\omega}{\omega_n}}+\frac{(j\omega)^2}{\omega_n^2}}$$

$\omega_n^2 = \frac{1}{LC}$ or $\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{4}$

$\omega RC = 2\zeta\frac{\omega}{\omega_n}$, $\zeta = \frac{\omega RC}{2\omega}\cdot\omega_n = \frac{\omega_n RC}{2}$

$= \frac{1\cdot 8}{4\times 2} = 1.0$

Critical damping, $\zeta = 1.0$ ....

Summarize

$K = 1.0$

$\zeta = 1.0$

$\omega_n = 1/4 \text{ r/s}$

Freq Response

CH 14

$s^2 + s \cdot \frac{R}{L} + \frac{1}{LC}$ ← Char EQ for series RLC

$\alpha = \frac{R}{2L} = \frac{1}{4}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

$\omega_0 = \omega_n$

$\alpha = \omega_0$ → Critical damping

CHAPTER 8