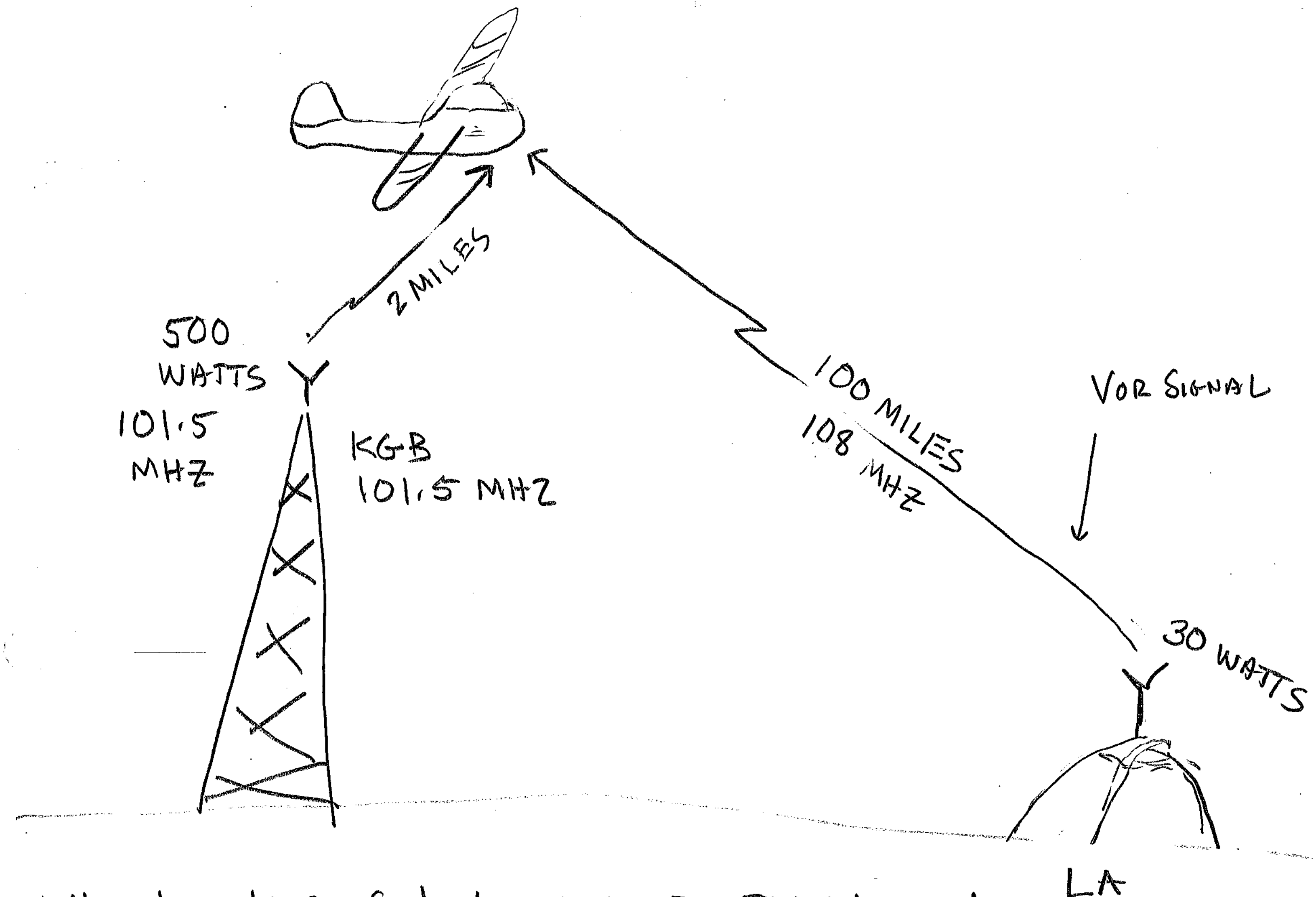
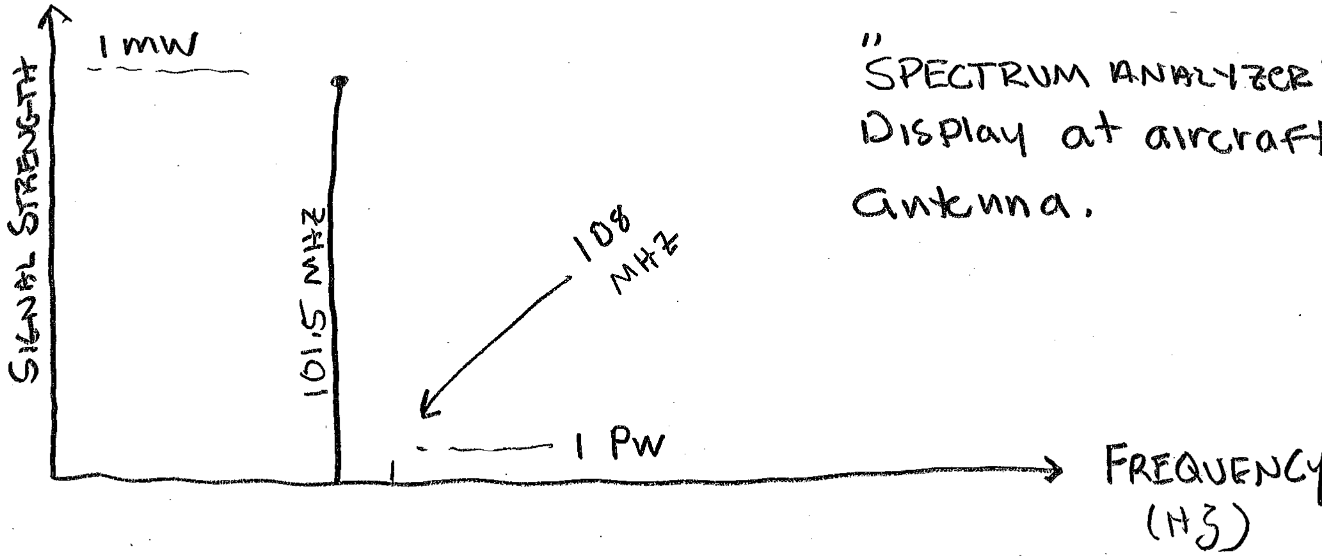
LECTURE 16 - INTRODUCTION TO FREQUENCY RESPONSE

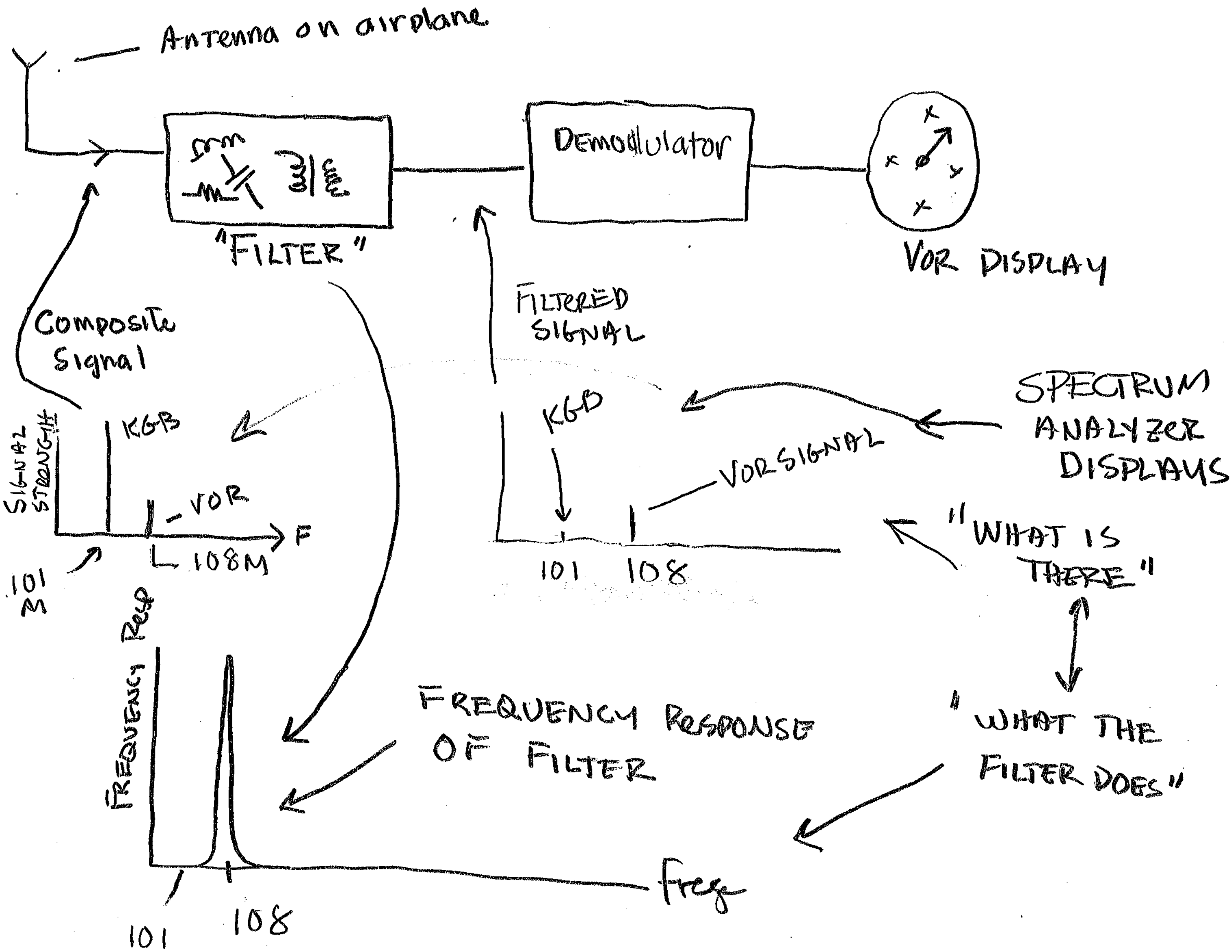


Why does KGB Get to OUTPUT 500 W and Air traffic control only get to OUTPUT 30 W ??

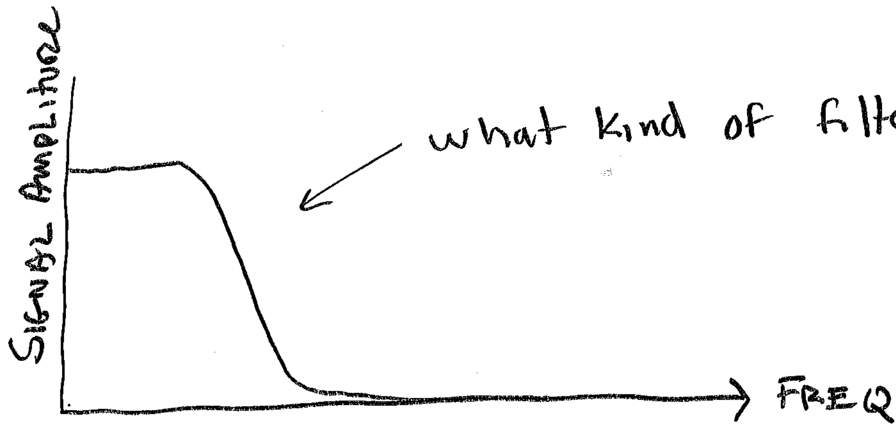


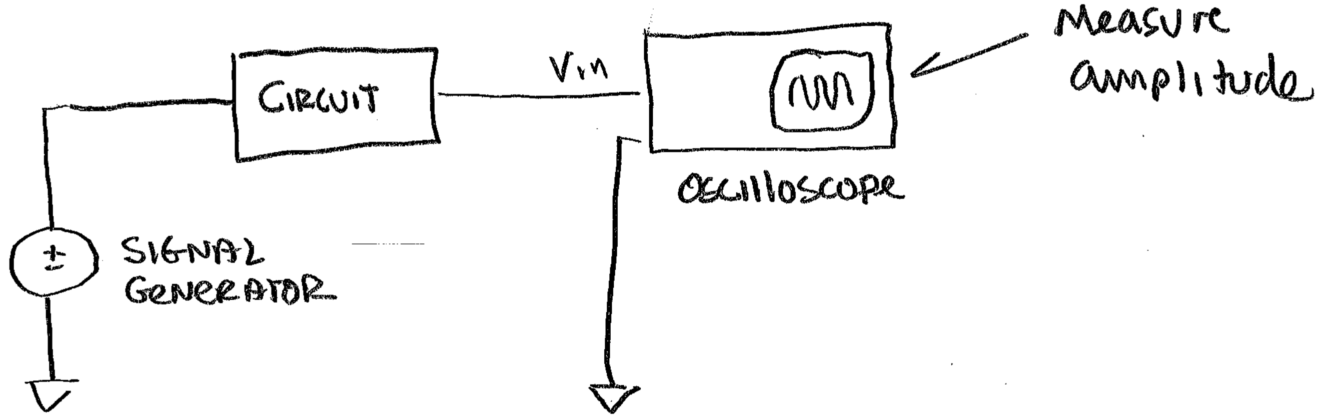
"SPECTRUM ANALYZER" Display at aircraft antenna.

$$\frac{1\ mw}{1\ pw} = 10^9$$ — That's a billion times more power!



The filter eliminates the interferer. The filter is described by it's "Frequency Response"

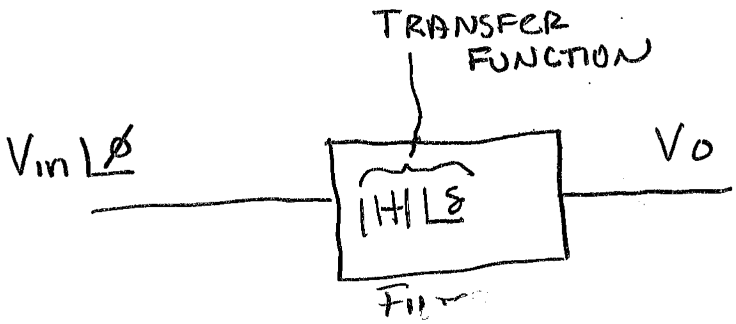




- Keep input level constant, vary Frequency
- Measure amplitude with oscilloscope.
- Impedance of inductors and capacitors changes with frequency. What would you expect?

From pg. 614

"The Transfer Function $H(\omega)$ of a circuit is the frequency-dependent ratio of a phasor OUTPUT, $Y(\omega)$ (an element voltage or current) to a phasor input $X(\omega)$ (source voltage or current)."



$$V_o = V_{in}\angle\phi \times |H|\angle\delta = V_{in}\cdot H \angle\phi+\delta$$

Connect to thuds and boings

$$r(t) = Ae^{(-\alpha+j\omega)t} = Ae^{-\alpha t}e^{j\omega t}$$

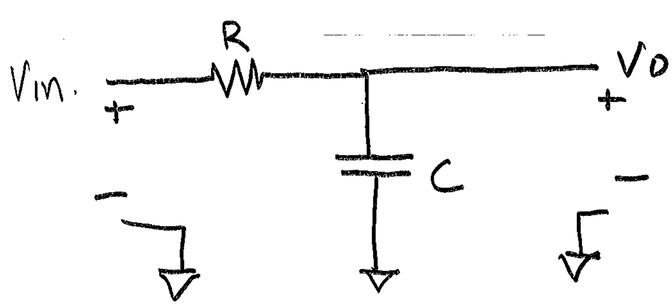
For sinusoidal steady state, $\alpha = 0$ so

$$r(t) = Ae^{j\omega t} = Ae^{st} \quad \text{where } s = j\omega$$

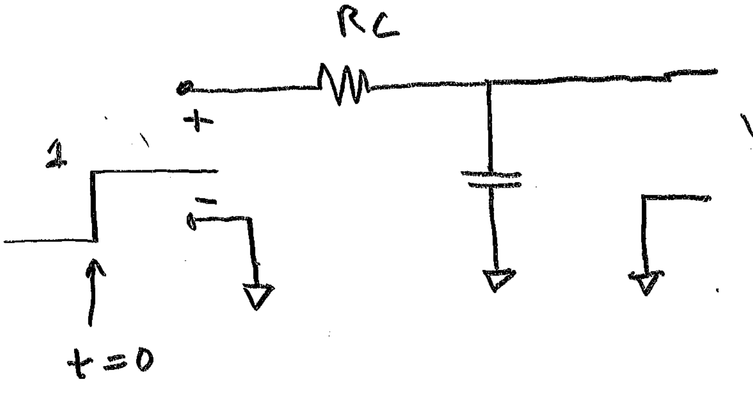
Goodbye $j\omega$... it's s now!

Example 14.1 Pg 615

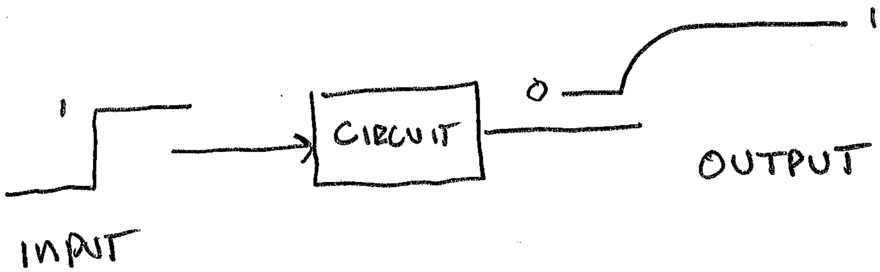
Find transfer function and frequency response of the RC circuit.



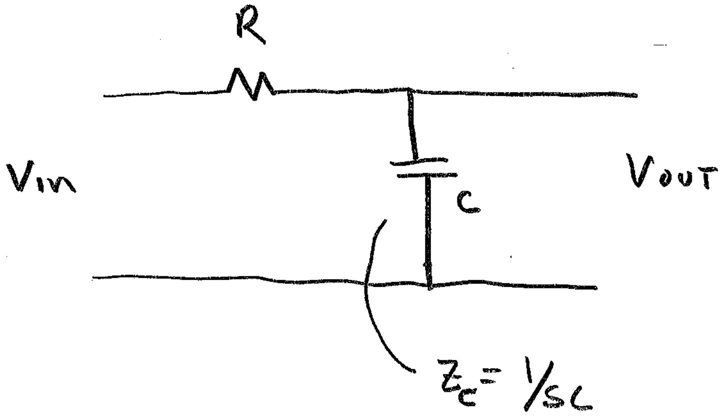
Before solving the right problem, let's solve a different one.



Input is u(t), OUTPUT is $V_o(t) = 1 - e^{-t/RC}$



Now solve the right problem

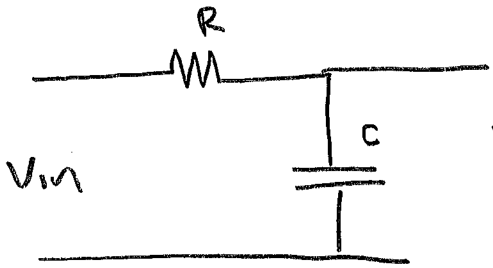


$$\frac{V_o}{V_{in}} = \frac{1/sC}{R + 1/sC} = \frac{1}{1 + sCR}$$

Introduce tool: "Asymptotic Analysis"

RC Circuit

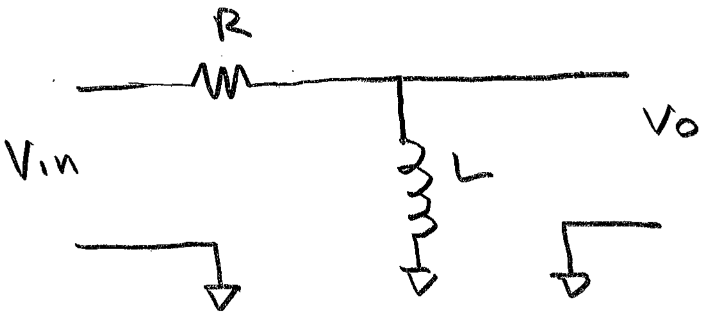
$$H(s) = H(j\omega) = \frac{1}{1+sCR}$$



Vo ← { Response at DC ?
Response at freq = ∞ ?

$$\frac{1}{1+sCR}$$ { Response at DC ?
Response at freq = ∞

This is an excellent way to see circuits intuitively and to check your work.



$$\frac{V_o}{V_{in}} = \frac{sL}{R+sL}$$

Circuit – Response at DC
Response at ∞

EQUATION – Response at DC
Response at ∞

Decibels

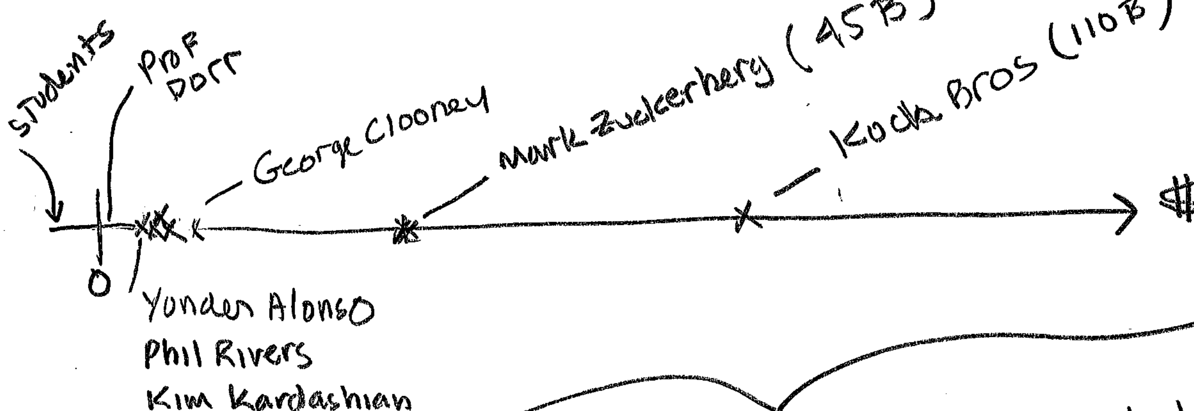
$$G = \text{Number of deci-bels} = 10 \log\left[\frac{P_2}{P_1}\right] \longleftarrow \text{Power gain}$$

$$G = \text{decibels} = 10 \log\left[\frac{V_2^2/R}{V_1^2/R}\right] = 20 \log\left(\frac{V_2}{V_1}\right)$$

Why do we need another term .....??!

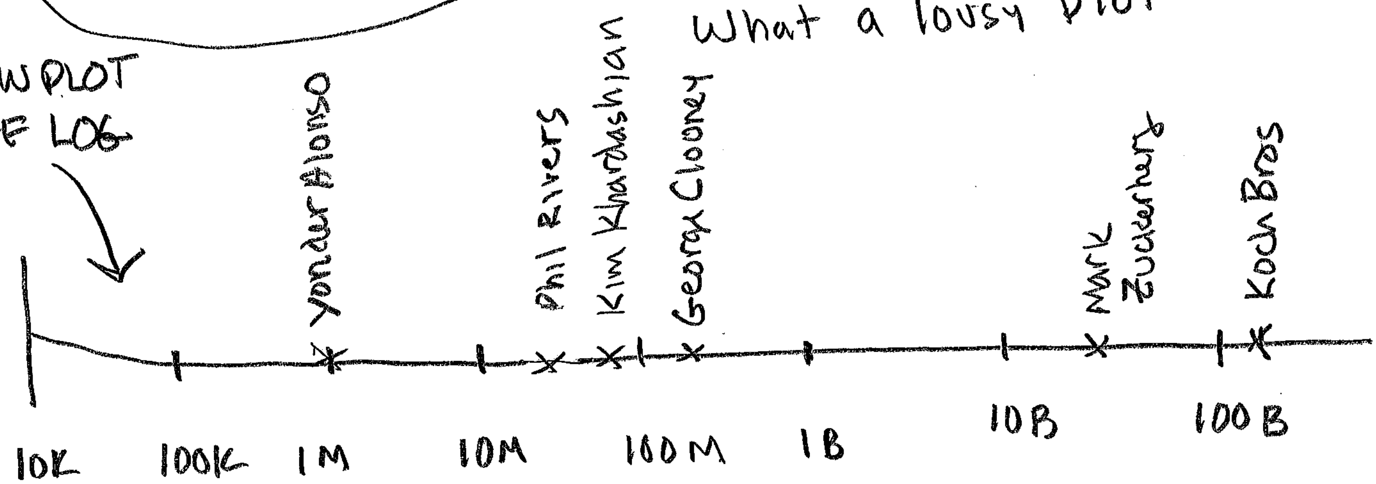
Who has the money?

Raise
\$1.00/hr Lin
or 10% Log



What a lousy plot

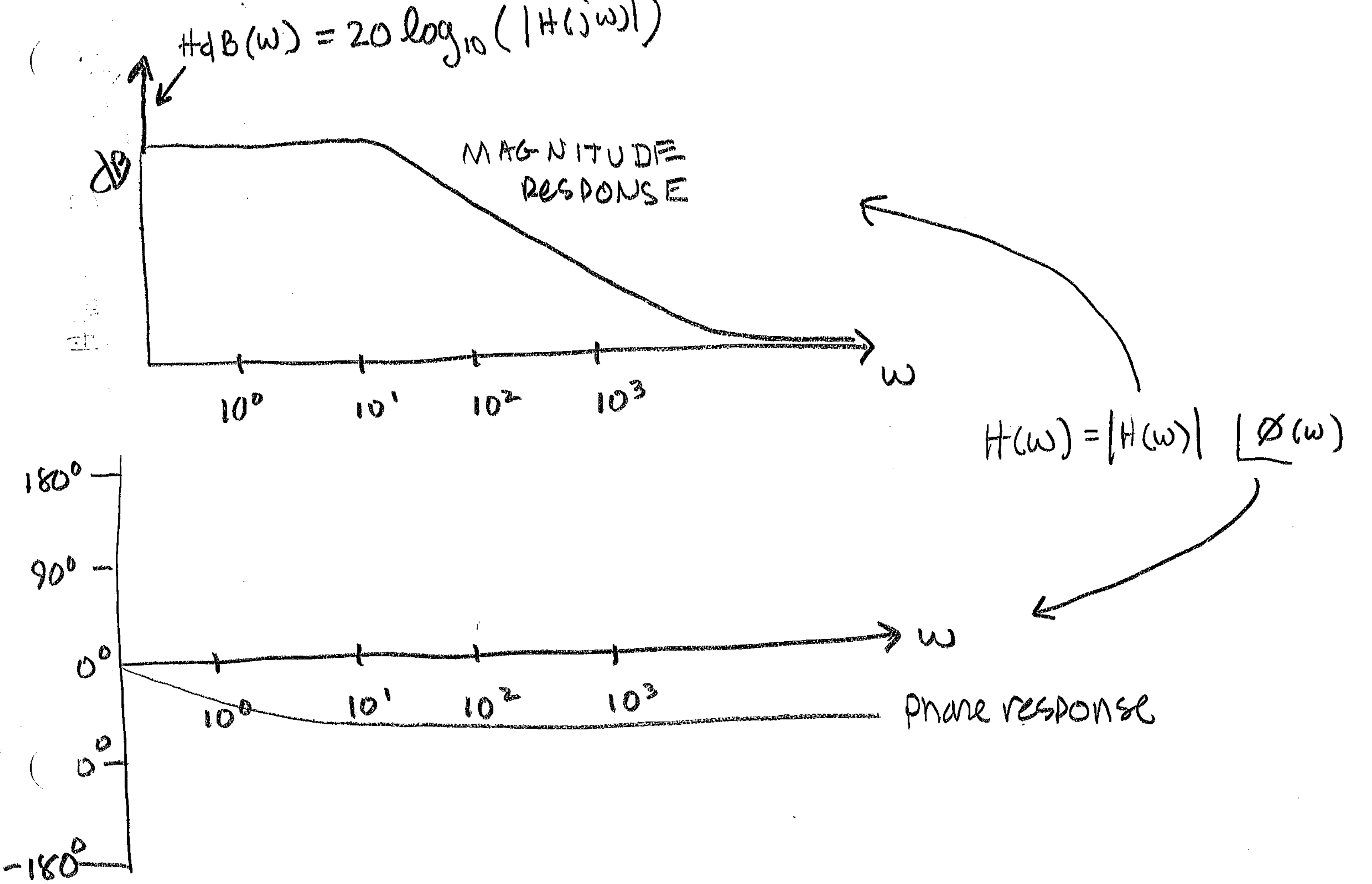
NOW PLOT THE LOG



Great Plot

Remember power ratio of 10^9 at beginning of lecture?

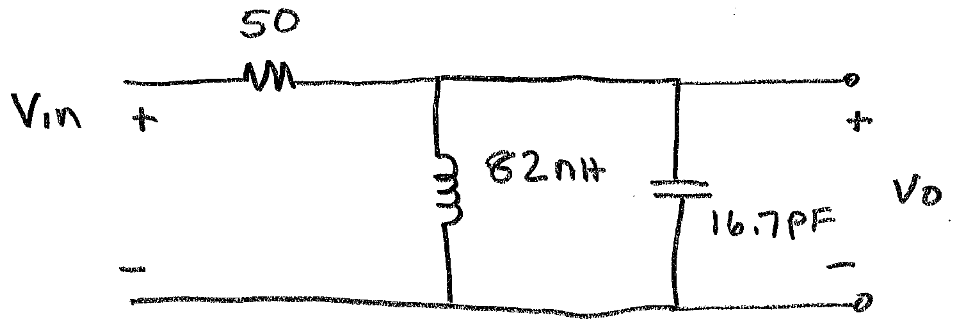
Plots for frequency response



FOR RC CIRCUIT,

$$H(s) = \frac{1}{1+sCR} = \frac{1}{1+j\omega RC} = \underbrace{\frac{1}{\sqrt{1+\omega^2R^2C^2}}}_{\text{MAG}} \; \underbrace{\angle -\tan^{-1}(\omega RC)}_{\text{phase.}}$$

Example - Find and plot Frequency Response

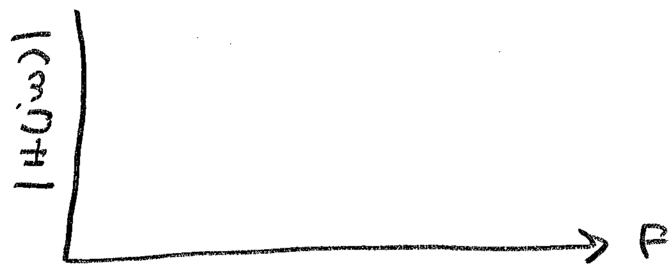


TRIAGE WITH Asymptotic analysis

Response at DC ?

Response at ∞ ?

Response in the middle ?



$$\frac{V_o}{V_{in}} = \frac{Z_L \| Z_C}{Z_L \| Z_C + R} =$$ WAIT, STOP, MAKE IT EASIER!

$$Z_L \| Z_C = \frac{SL \times \frac{1}{SC}}{SL + 1/SC} = \frac{SL}{S^2LC + 1} = \frac{N}{D}$$

$$\text{so } \frac{V_o}{V_{in}} = \frac{\frac{N}{D}}{\frac{N}{D} + R} = \frac{N}{N + DR} = \frac{SL}{SL + (S^2LC + 1)R}$$

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{sL}{s^2 RLC + R + sL} = \frac{\frac{sL}{RLC}}{s^2 \frac{RLC}{RLC} + \frac{R}{RLC} + \frac{sL}{RLC}}$$

$$H(s) = \frac{1}{RC} \times \frac{s}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

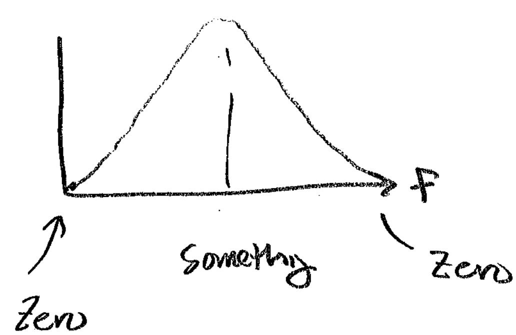
Hey, note the similarity to parallel RLC characteristic equation

Check it . . .

DC $\rightarrow$ $s = 0$ $\rightarrow$ $H(s) = \frac{1}{RC} \cdot \frac{0}{1/LC} = 0$ ✓

freq $= \infty$, $s = \infty$ $H(s) = \frac{\infty}{\infty^2} = 0$ ✓

mid range $\rightarrow$ something



LOGARITHMIC COUNTING

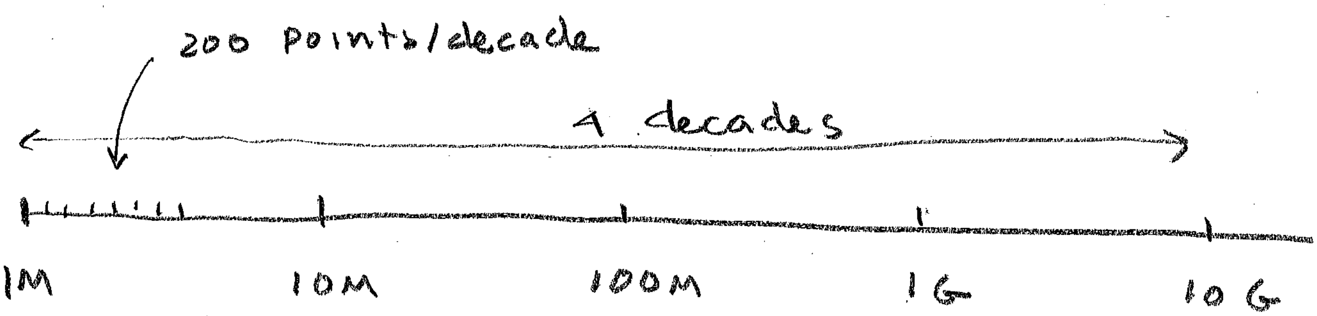
PROBLEM: Create a set of frequency values that are uniformly spaced on a log plot.

- Start at frequency F_s
- Plot covers d decades
- Plot ppd points per decade

Then the plot contains $d \times ppd$ points and the frequencies are:

$$F_K = F_s \times 10^{\frac{K}{ppd}} \quad , K = 0, 1, \ldots (d \times ppd - 1)$$

Say $F_s = 1$ Mhz, $d = 4$ decades, $ppd = 200$ points/dec



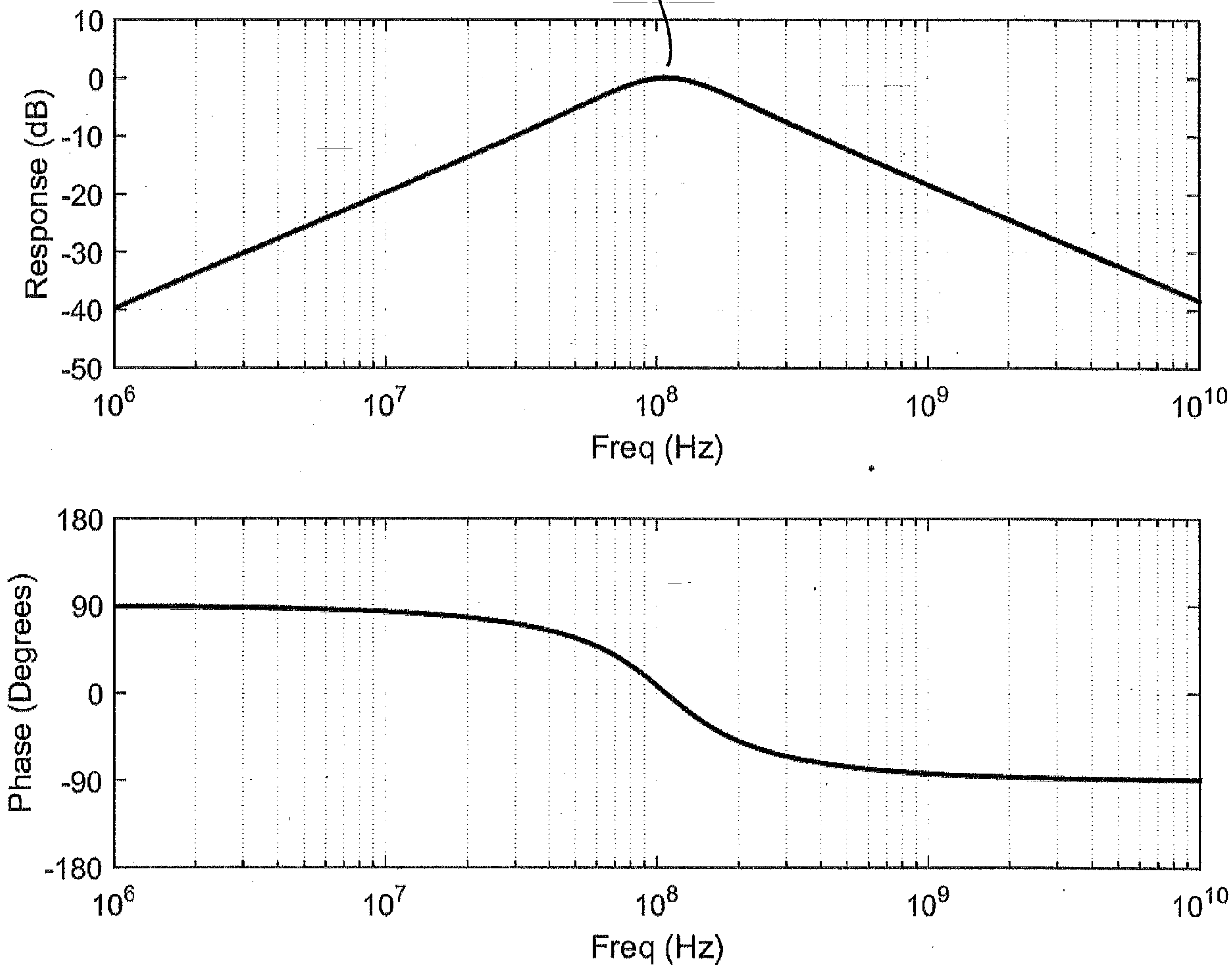
Note: when $K > ppd$,

$$F_K = F_s \times 10^{\frac{ppd + K'}{ppd}} = F_s \times 10^{\frac{ppd}{ppd}} \times 10^{\frac{K'}{ppd}}$$

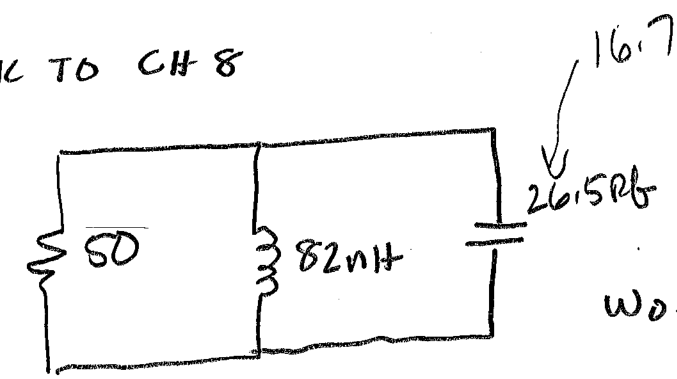
$$= 10 F_s \times 10^{K'/ppd}$$

... and so on ...

Bandpass centered at 108 Mhz



FLASHBACK TO CH 8



$\alpha = 3.77e8$, $\omega_0 = 6.78e8$

$\omega_0 > \alpha$ so underdamped

$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow 6.78e8$ rad/sec

That's 108 Mhz

Remember corner pipe!

```
% Filename:          ComputeFreqResp.m
% Author:            Barry Dorr
% Last Modified:     10-23-2012

% This script computes the frequency response for a s-domain transfer function.

clear all;
close all;

StartFreq = 1000000;     % Lowest frequency to plot.
NumDec = 4;          % Number of decades to plot.
PtsPerDec = 200;     % Number of frequency points plotted per decade.

% Pre-allocate results matricies.
MagResp = zeros(NumDec*PtsPerDec,1);     % Matrix containing magnitude response.
PhaseResp = zeros(NumDec*PtsPerDec,1);   % Matrix containing phase response.
Freq = zeros(NumDec*PtsPerDec,1);        % Matrix containing plot frequencies.

% Values for example
R = 50;
L = 82e-9;
C = 26.48e-12;

% Compute frequency response at frequency points uniformly-spaced on a log plot.
for i=1 : NumDec*PtsPerDec
    Freq(i) = StartFreq*10^(i/PtsPerDec);    % Evaluate at this frequency.
    s = 1j * 2*pi*Freq(i);                   % Determine complex frequency.

    H = 1/(R*C) * s/(s^2 + s/(R*C) + 1/(L*C));  % Compute response at this frequency.

    MagResp(i) = abs(H);                     % Place magnitude response in result matrix.
    PhaseResp(i) = angle(H)*360/(2*pi);      % Place phase response in result matrix.
end


% Plot magnitude and phase.
subplot(2,1,1);
HMag=semilogx(Freq,20*log10(MagResp), 'color','k', 'linewidth', 2); % Plot magnitude
set(gca,'FontSize',12);
xlabel('Freq (Hz)');
ylabel('Response (dB)');
ylim([-50 10]);
grid on;

subplot(2,1,2);
HPhs = semilogx(Freq,PhaseResp, 'color','k', 'linewidth', 2); % Plot phase
set(gca,'FontSize',12);
xlabel('Freq (Hz)');
ylabel('Phase (Degrees)');
ylim([-180 180]);
set(gca,'YTick',[-180, -90, 0, 90, 180]);
grid on;
```