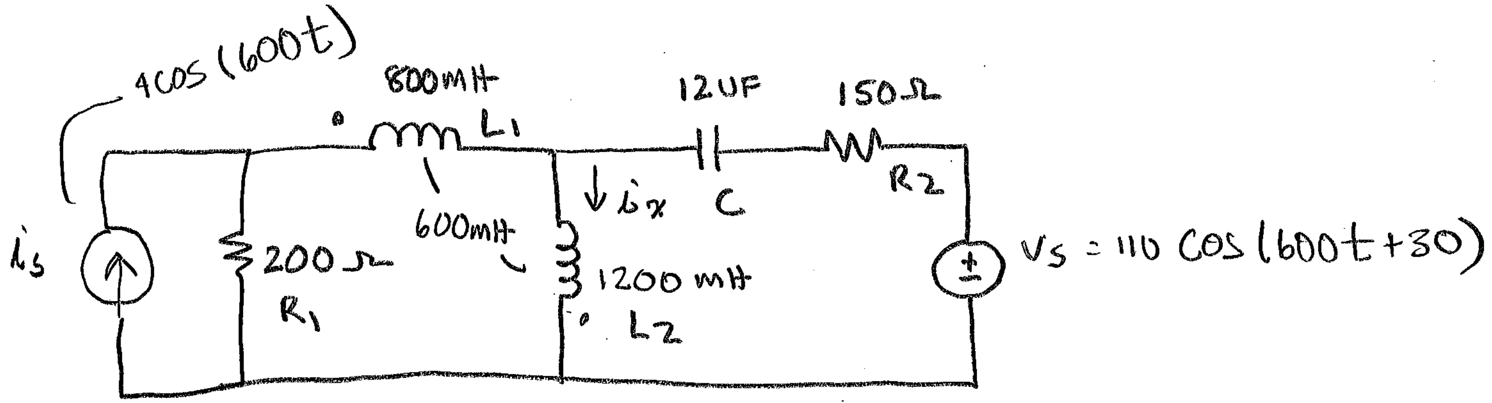
LECTURE 15 - FUN WITH TRANSFORMERS

3.11 Find i_x Using mesh analysis

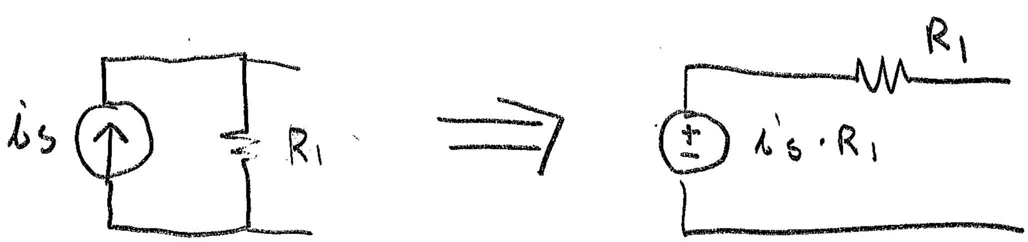


The coils are coupling, but a lot or just a bit?

we know $M \leq \sqrt{L_1 L_2}$ EQ 13.35

and
$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.6}{\sqrt{0.8 \times 1.2}} = 0.6124$$

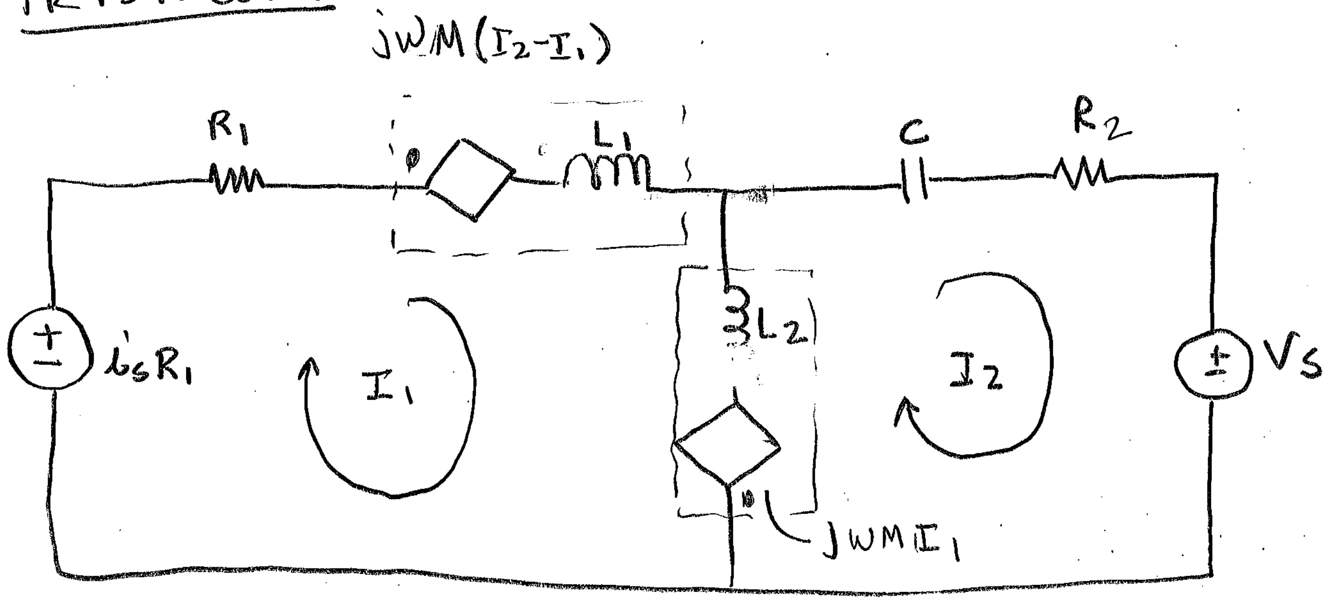
for mesh analysis we'll use source transformation



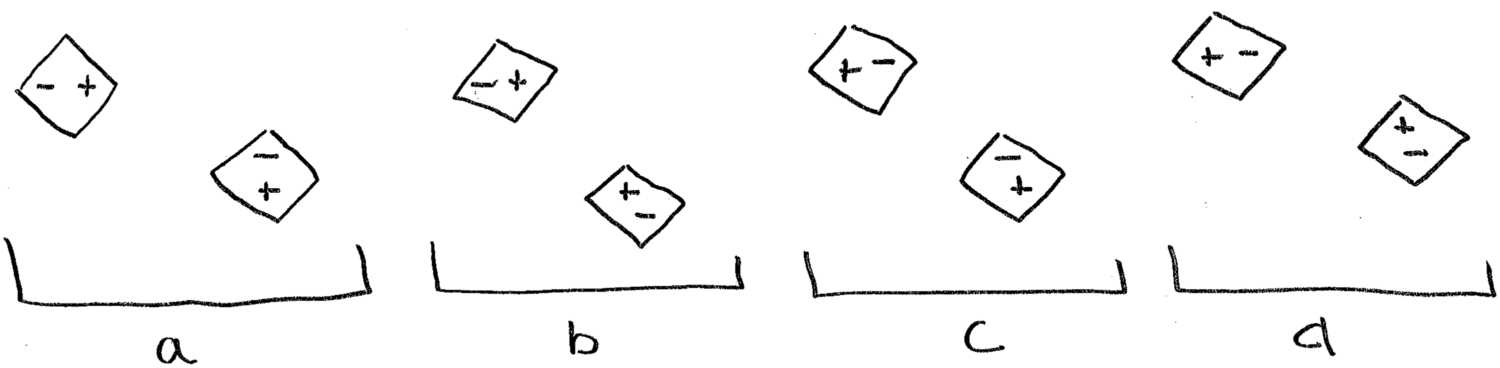
$i_s = 4\cos(600t)$ ⟶ what is Freq in Hz?

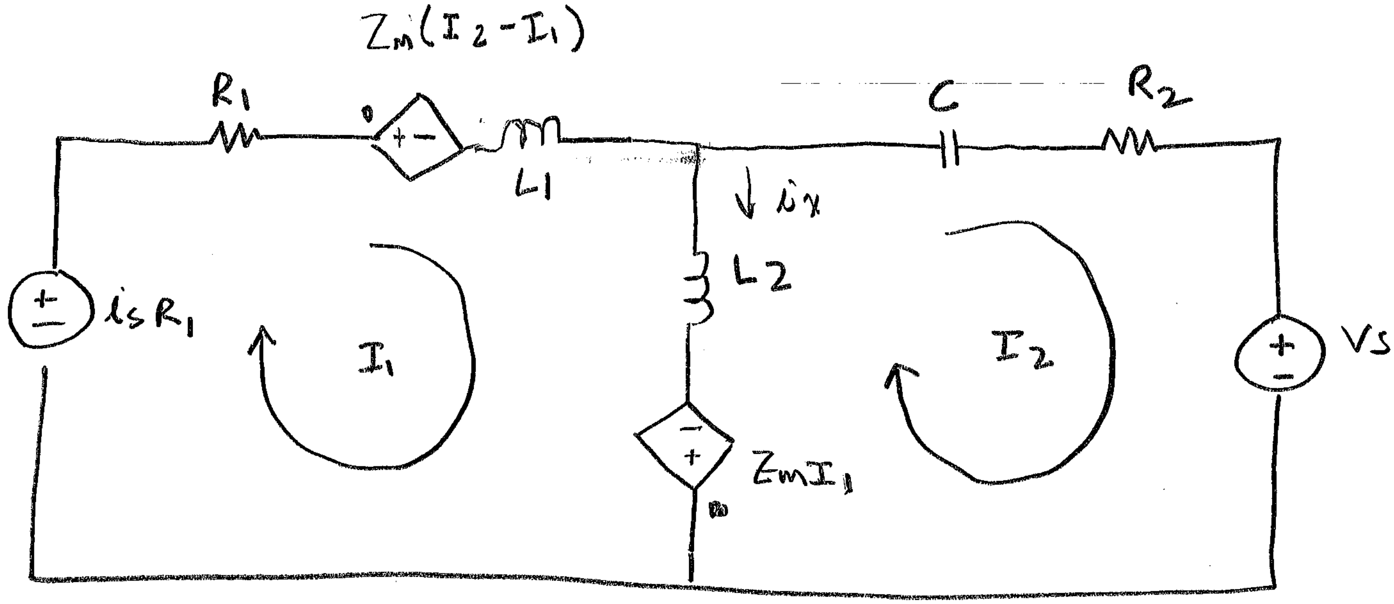
a - 600 Hz
b - 3769 Hz
c - none of the above

PR 13.11 CONT



"It is . . . time to vote"





(1) $-I_sR_1 + I_1R_1 + Z_M(I_2-I_1) + Z_{L1}I_1 + Z_{L2}(I_1-I_2) - Z_mI_1 = 0$

$I_1(R_1 - Z_m + Z_{L1} + Z_{L2} - Z_m) + I_2(Z_m - Z_{L2}) = I_sR_1$

(2) $Z_mI_1 + (I_2-I_1)Z_{L2} + I_2Z_C + I_2R_2 + V_s = 0$

$I_1(Z_m - Z_{L2}) + I_2(Z_{L2} + Z_C + R_2) = -V_s$

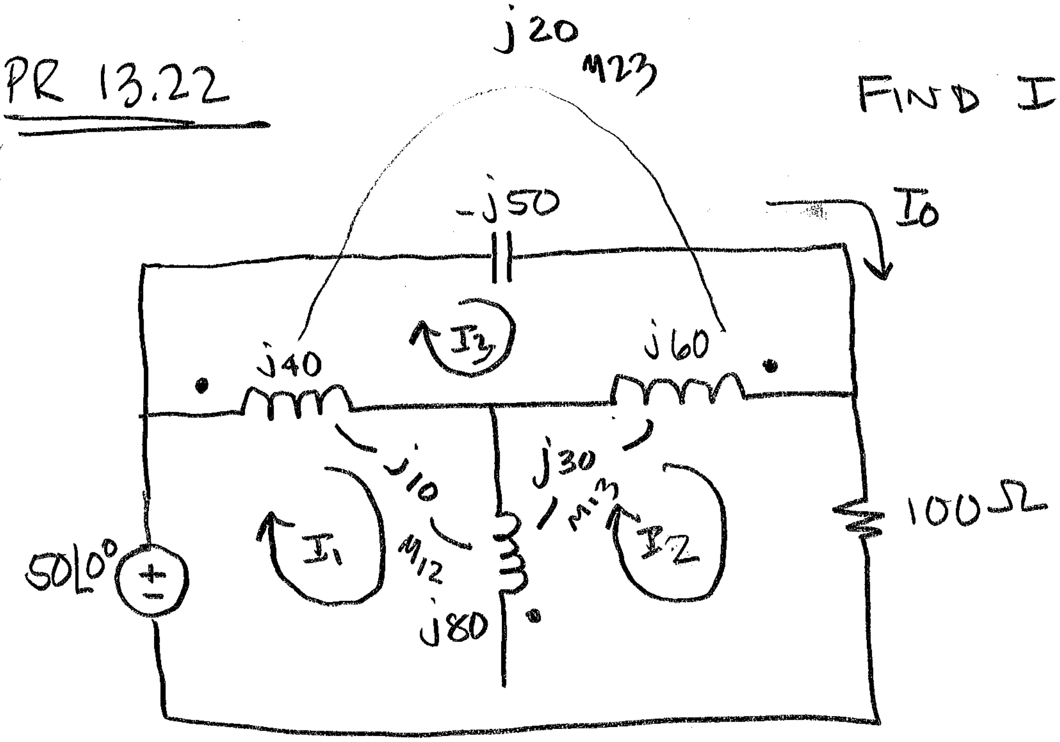
$$\begin{bmatrix} R_1 - Z_m + Z_{L1} + Z_{L2} - Z_m & Z_m - Z_{L2} \\ Z_m - Z_{L2} & Z_{L2} + Z_C + R_2 \end{bmatrix}^{-1} \begin{bmatrix} I_sR_1 \\ -V_s \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

and $I_x = I_1 - I_2 = 1.07\cos(600t - 66^\circ)$

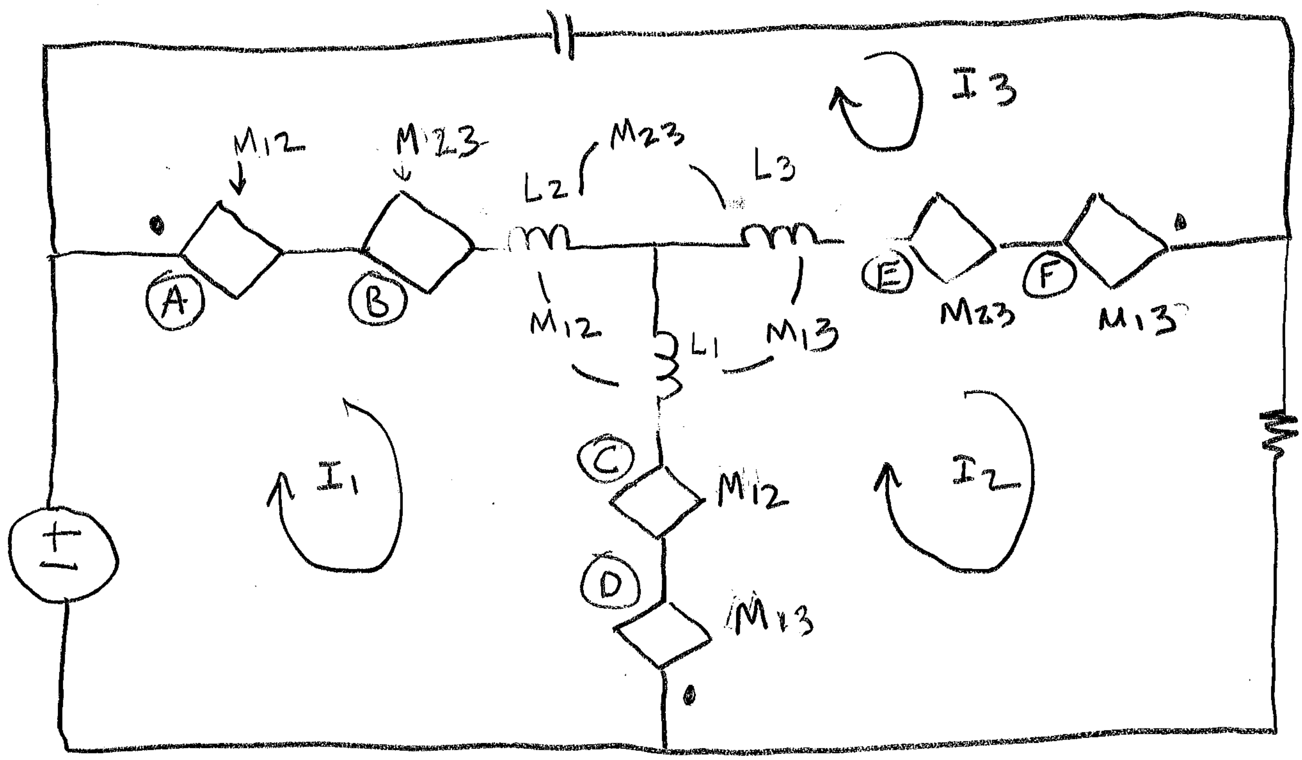
I disagree with answer in back of book

PR 13.22

FIND Io



"Bridged - T" network



made current entering dot of "donor" Positive

(A) → $SM_{12}(I_2 - I_1)$

(B) → $SM_{23}(I_3 - I_2)$

(C) → $SM_{12}(I_1 - I_3)$

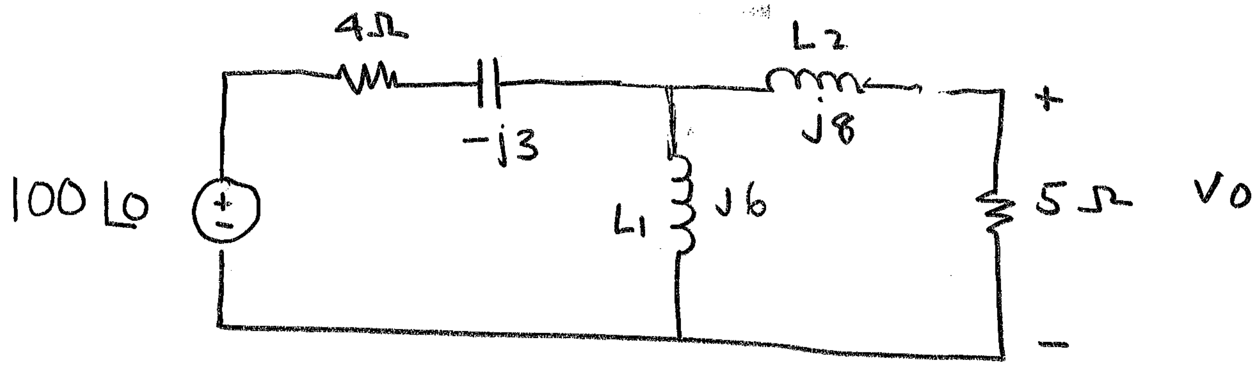
(D) → $SM_{13}(I_3 - I_2)$

(E) → $SM_{23}(I_1 - I_3)$

(F) → $SM_{13}(I_2 - I_1)$

FILL IN THE POLARITY ON THE SOURCES

Practical problem

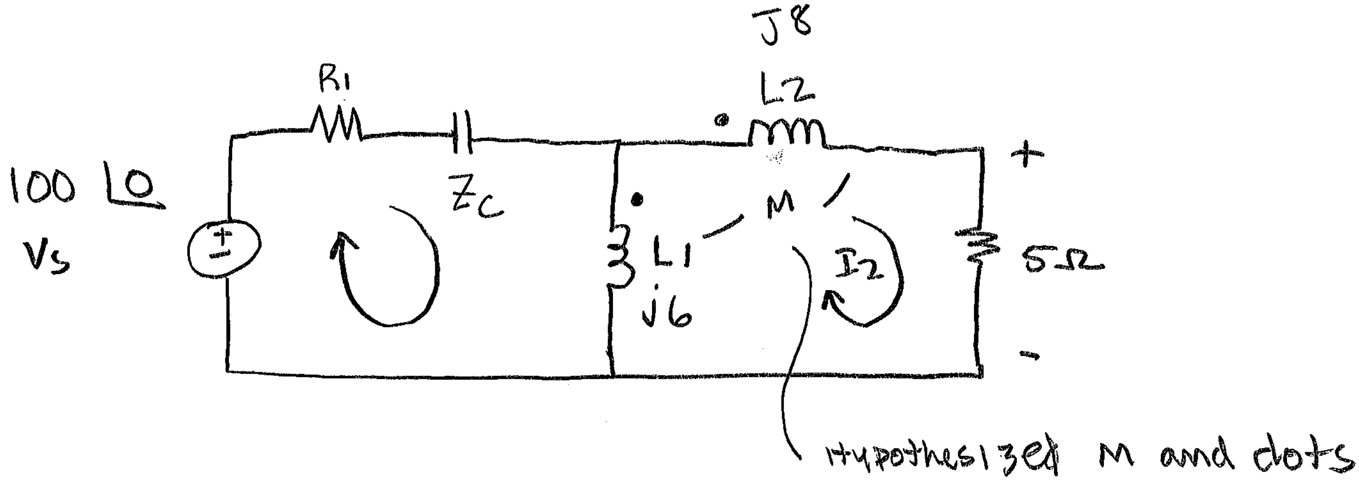


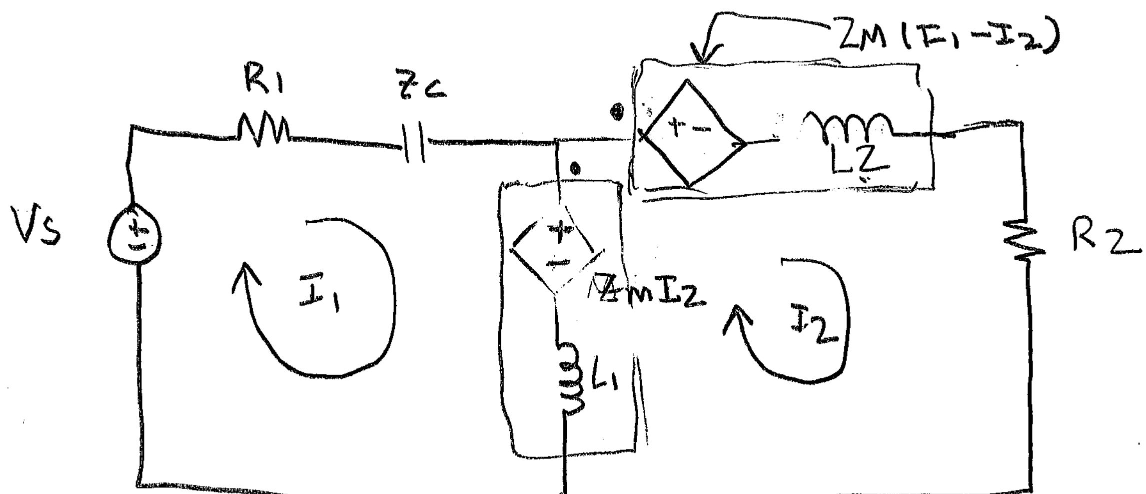
1) "There is no mutual inductance." Compute V_o (design value)
2) Measure the OUTPUT voltage. See if it equals the design value.

4) If the OUTPUT voltage doesn't match the design value, compute the mutual inductance.

Strategy

1) Look at the circuit. Hypothesize where dots are
2) Compute the expected OUTPUT with $M = 0$. This is the design value.
3) Measure the OUTPUT voltage
4) Adjust M in equations until we match the measured voltage





I_2 enters dot of L_2 so it makes dot of L_1 more positive

I_1 enters dot of L_1 so it makes dot of L_2 more positive

└ Note $(I_1 - I_2) Z_M$ for L_2

① $-V_S + R_1 I_1 + Z_C I_1 + Z_M I_2 + (I_1 - I_2) Z_{L1} = 0$

$I_1 (R_1 + Z_C + Z_{L1}) + I_2 (+Z_M - Z_{L1}) = V_S$

② $(I_2 - I_1) Z_{L1} - Z_M I_2 + I_2 Z_{L2} + Z_M (I_1 - I_2) + I_2 R_2 = 0$

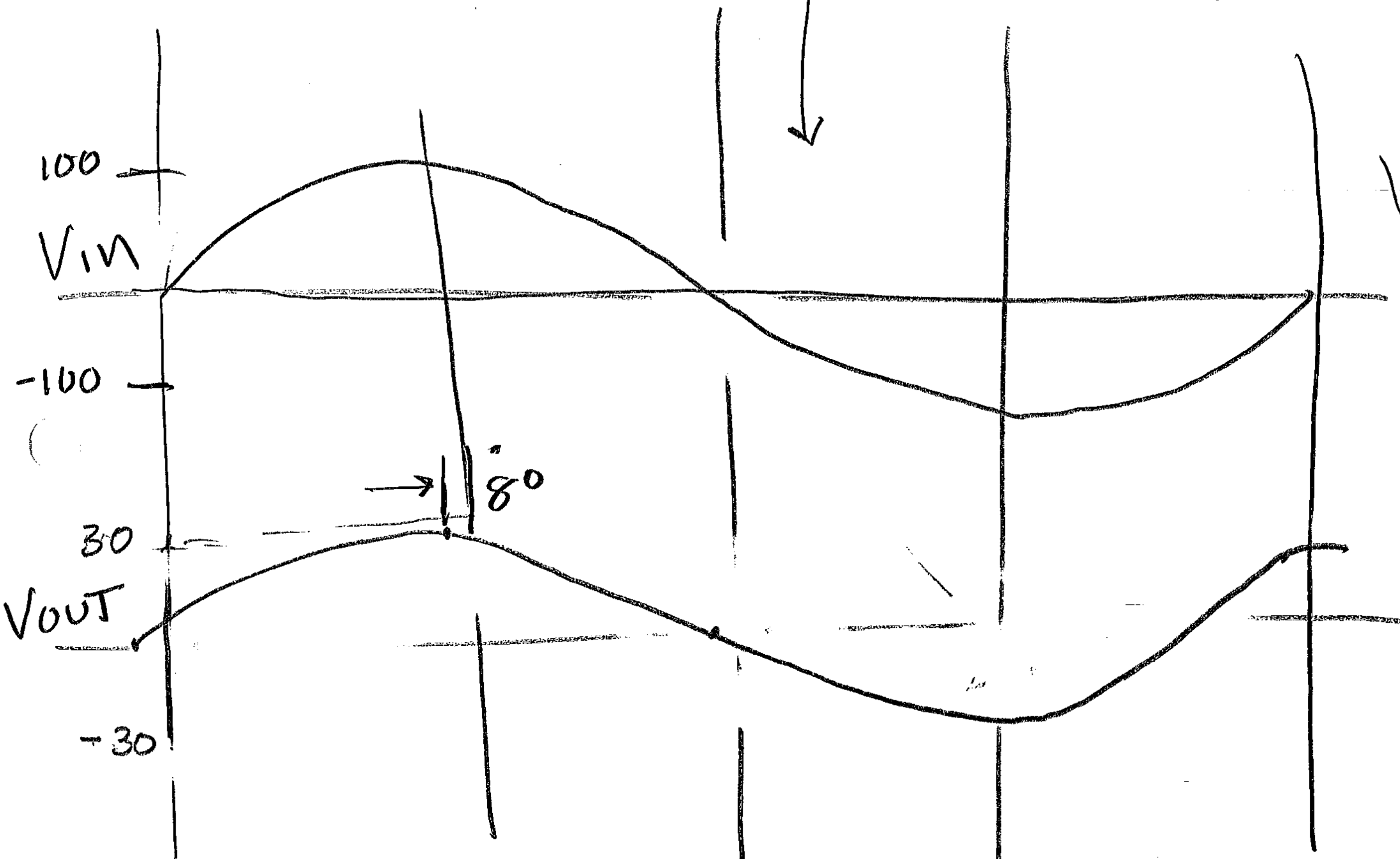
$I_1 (-Z_{L1} + Z_M) + I_2 (Z_{L1} - Z_M + Z_{L2} - Z_M + R_2) = 0$

$$\begin{bmatrix} R_1+Z_C+Z_{L1} & +Z_M-Z_{L1} \\ -Z_{L1}+Z_M & Z_{L1}-Z_M+Z_{L2}-Z_M+R_2 \end{bmatrix}^{-1} \begin{bmatrix} V_S \\ 0 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

WITH $Z_M=0$, NO COUPLING

$V_0 = I_2 R_2 = (8.134 + j1.60) \times 5 \quad 40.67 + j8.02 \rightarrow 41.45 \angle 11.15^\circ$

MEASURE WAVEFORM BELOW



Is the circut working right?

So what do we do?

1) remove the components and measure them

- They are ok

2) Hypothesize mutual coupling.

- Note desired phase was 11.15°, we got about 8°. That's pretty hard to measure.

- But we expected amplitude of 41 and we got about 27. That's easy to see.

3)

Z_M	$\lvert V_o \rvert$	$\angle$ phase	
j1	39.3	8.1°	
j2	36.1	6.2°	
j3	31.7	6.07°	
j4	25.34	8.740	← Too far
j3.77	27.04	7.79	← Matches what we saw

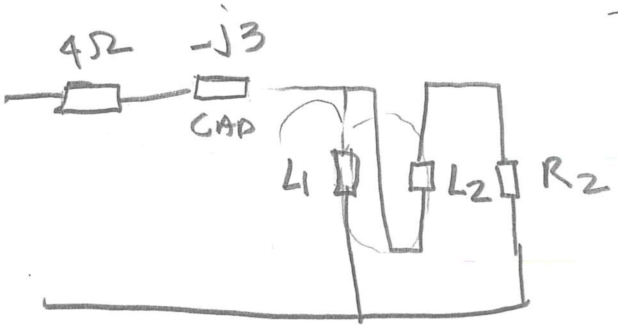
can I do this?

$$K = \text{Coefficient of Coupling} = \frac{M}{\sqrt{L_1 L_2}} \stackrel{?}{=} \frac{j\omega M}{\sqrt{j\omega L_1 \cdot j\omega L_2}}$$

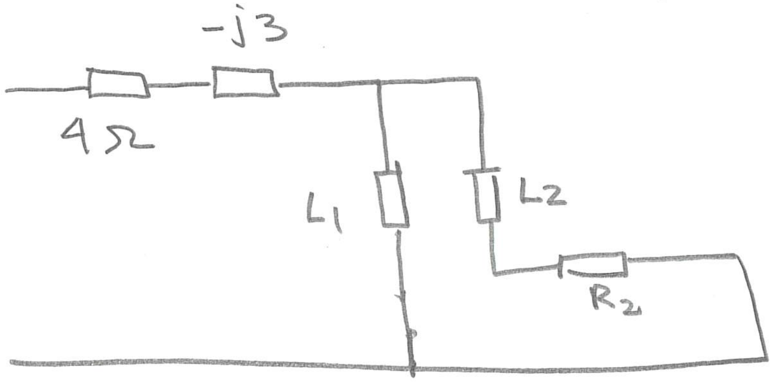
yes

$$= \frac{j\omega M}{\sqrt{-1\omega^2 L_1 L_2}} = \frac{j\omega M}{j\omega \sqrt{L_1 L_2}} = \frac{M}{\sqrt{L_1 L_2}}$$

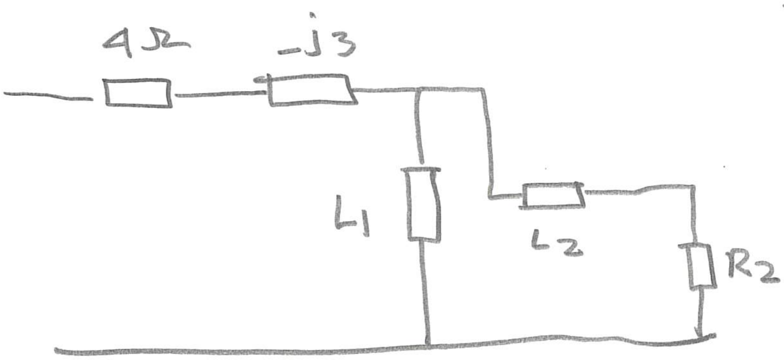
So $K = \dfrac{j3.77}{\sqrt{j6 \times j8}} = \underline{0.5442}$



Found this layout

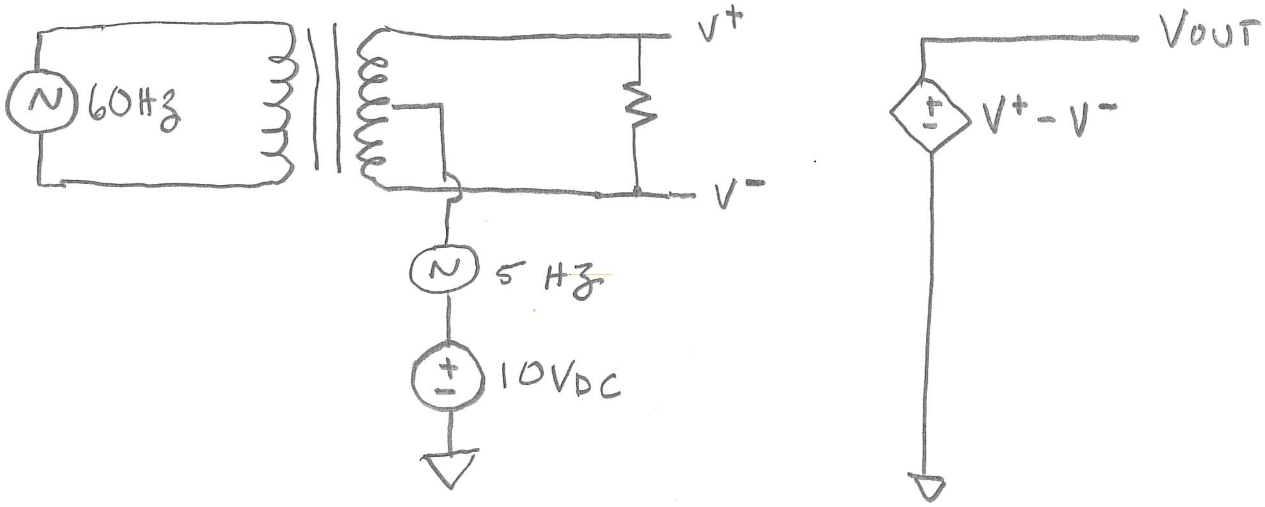


What will this do?



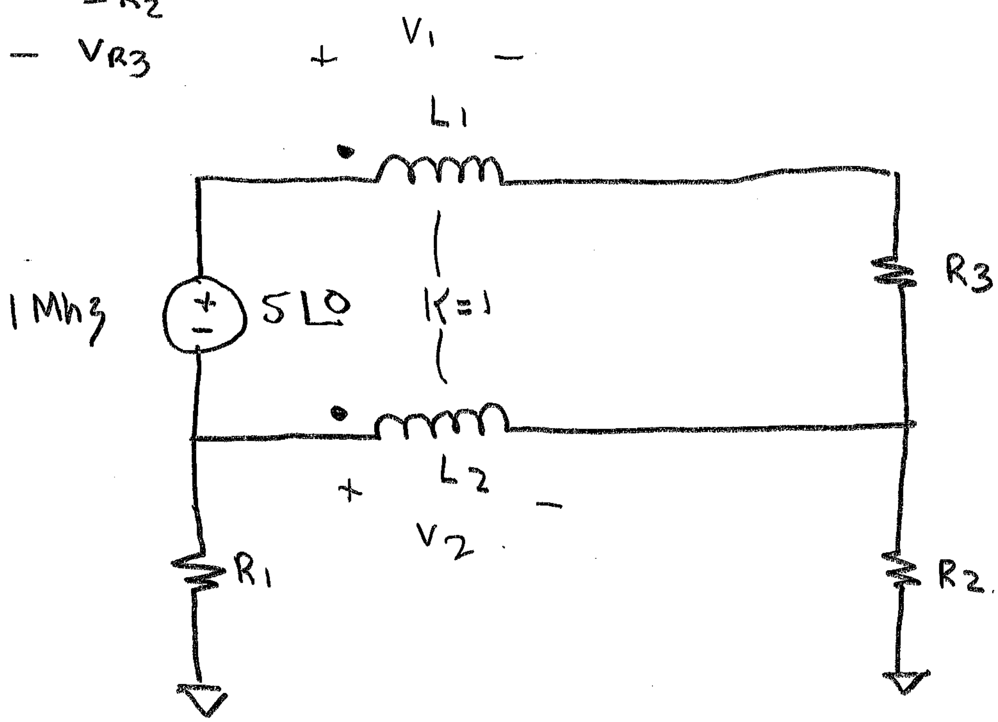
Fixed it!

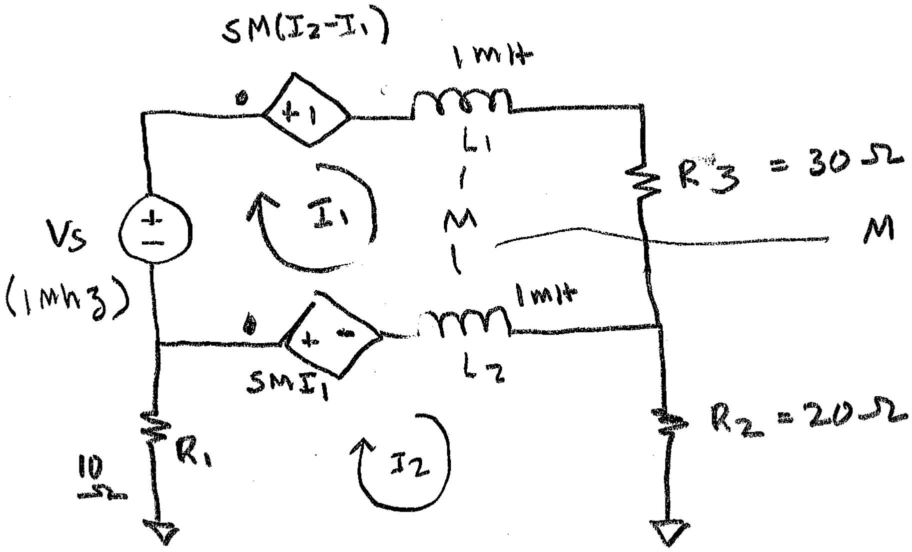
Center Tapped Transformer



LONGITUDINAL TRANSFORMERS

FIND
- V_1
- V_2
- I_{R_1}
- I_{R_2}
- V_{R_3}





$M = K\sqrt{L_1 L_2}$ EQ 13.37

① $-V_S + SM(I_2 - I_1) + SL_1 I_1 + R_3 I_1 + SL_2(I_1 - I_2) - SMI_1$

① $-V_S + SMI_2 - SMI_1 + SL_1 I_1 + I_1 R_3 + SL_2 I_1 - SL_2 I_2 - SMI_1 = 0$

① $(-SM + SL_1 + R_3 + SL_2 - SM)I_1 + (SM - SL_2)I_2 = V_S$

② $I_2 R_1 + SMI_1 + SL_2(I_2 - I_1) + I_2 R_2 = 0$

② $(SM - SL_2)I_1 + (R_1 + SL_2 + R_2)I_2 = 0$

Substitute vales - $S = j2\pi \times 1\text{Mhz}$, $R_1 = 10\,\Omega$, $R_2 = 20\,\Omega$, $R_3 = 30\,\Omega$

$L_1 = L_2 = 1\,\text{mH}$

$$\begin{bmatrix} -SM + SL_1 + R_3 + SL_2 - SM & SM - SL_2 \\ SM - SL_2 & R_1 + SL_2 + R_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_S \\ 0 \end{bmatrix}$$

Say $L_1 = L_2 = 1\,\text{mH}$, $K = 1$ Then $I_1 = 167\,\text{mA}$, $I_2 = 0$

$V_{L1} = SM(I_2 - I_1) + SL_1 I_1 = 0$

$V_{L2} = SMI_1 + SL_2(I_2 - I_1) = 0$

$V_{R3} = R_3 \times I_1 = 5V \longrightarrow$ Hey, thats V_S

$V_{R2} = I_2 R_2 = 0$, $V_{R1} = I_2 R_1 = 0$

in the previous example we note that

1) $K = 1$

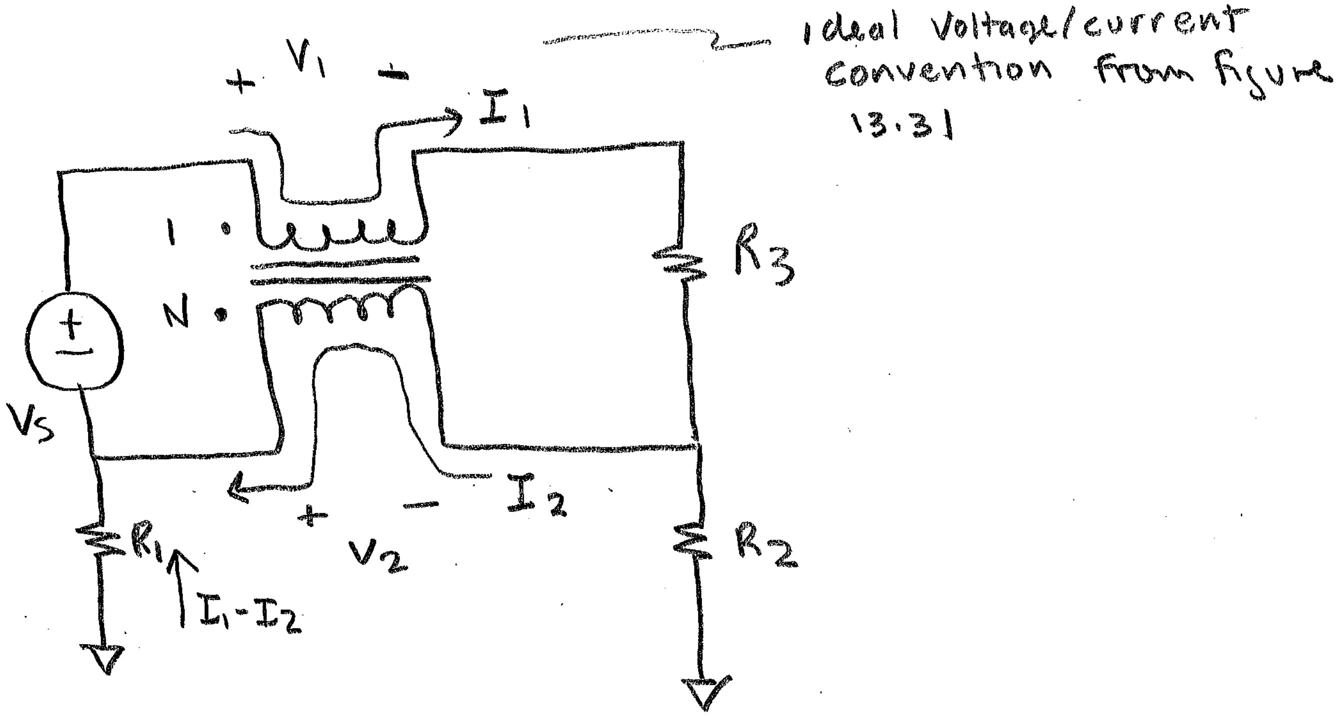
2) impedance of inductors $= j2\pi f L = j6.28\,K\Omega$
is much larger than the resistances.

Can we use ideal transformer model?

We have L_1 and L_2, what is turns ratio N?

See top of pg 574

$$n = \sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{1mH}{1mH}} = 1.0$$



① upper mesh

$-V_S + V_1 + I_1 R_3 - V_2 = 0$

② lower mesh

$(I_1 - I_2) R_1 + V_2 + (I_1 - I_2) R_2 = 0$

2 EQ 4 unknowns

Ideal transformer: $V_2 = N V_1$, $I_2 = I_1 / N$

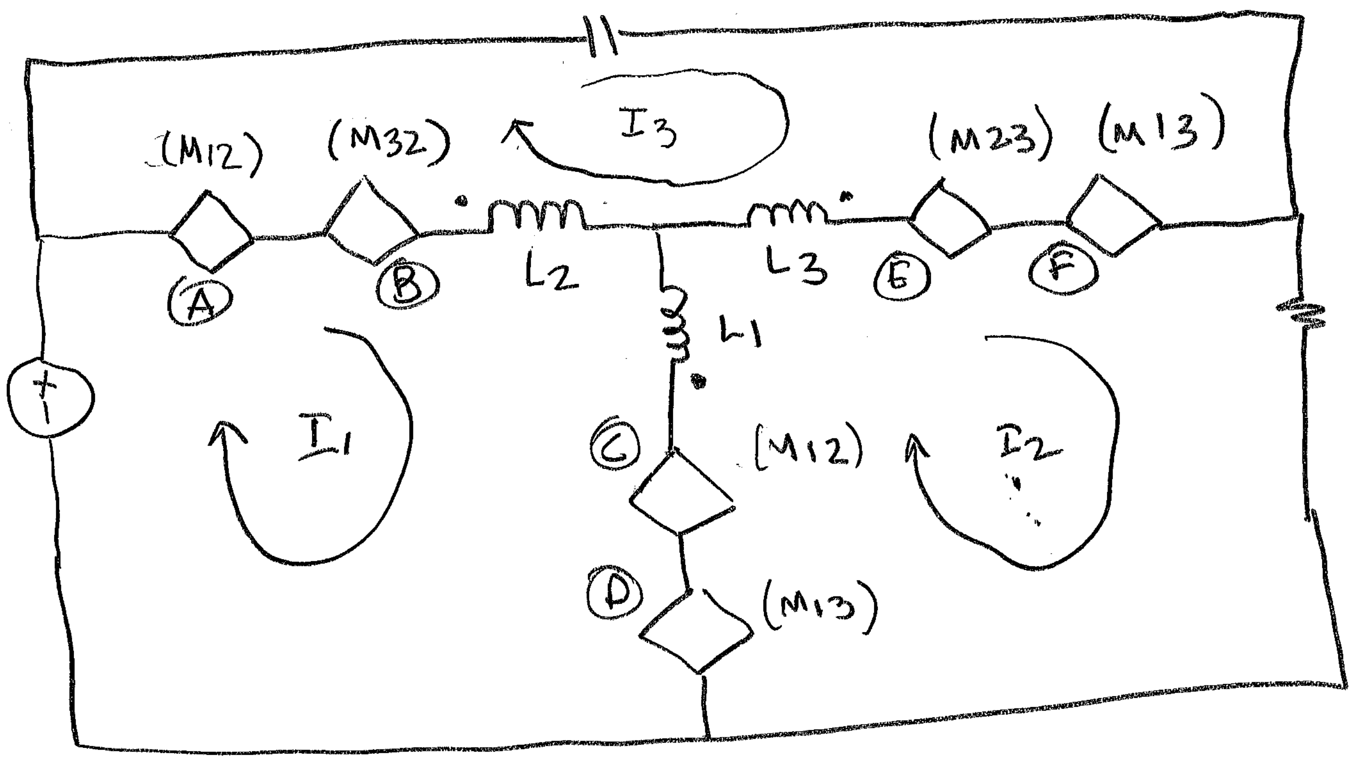
Since $N = 1$, $I_1 = I_2$

Therefore by KCL, $I_{R_1} = I_{R_2} = 0$

This is called a "Common Mode choke"

V_S is seen at R_3 Regardless of the values of R_1 and R_2!

Professor Dorr's secret notes For Problem 13.22



I define current so it enters dot

(A) → $I_{L_1} = I_2 - I_1$, I_2 makes DOT of L_2 more POS

$\diamond$ (+ −) $sM_{12}(I_2 - I_1)$

(B) → $I_{L3} = I_3 - I_2$, I_3 makes dot of L_2 more positive

$\diamond$ (+ −) $sM32(I_3 - I_2)$

(C) → $I_{L2} = I_1 - I_3$ I_1 makes Dot of L_1 more pos

$\diamond$ (− +) $sM_{12}(I_1 - I_3)$

Professor Dorr's secret notes

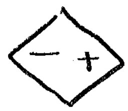
LB

(D) $I_{L3} = I_3 - I_2$ $\quad$ I_3 makes dot of L_1 more pos

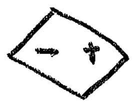
so

 $SM_{13}(I_3 - I_2)$

(E) $I_{L2} = I_1 - I_3$, I_1 makes dot of L_3 more pos

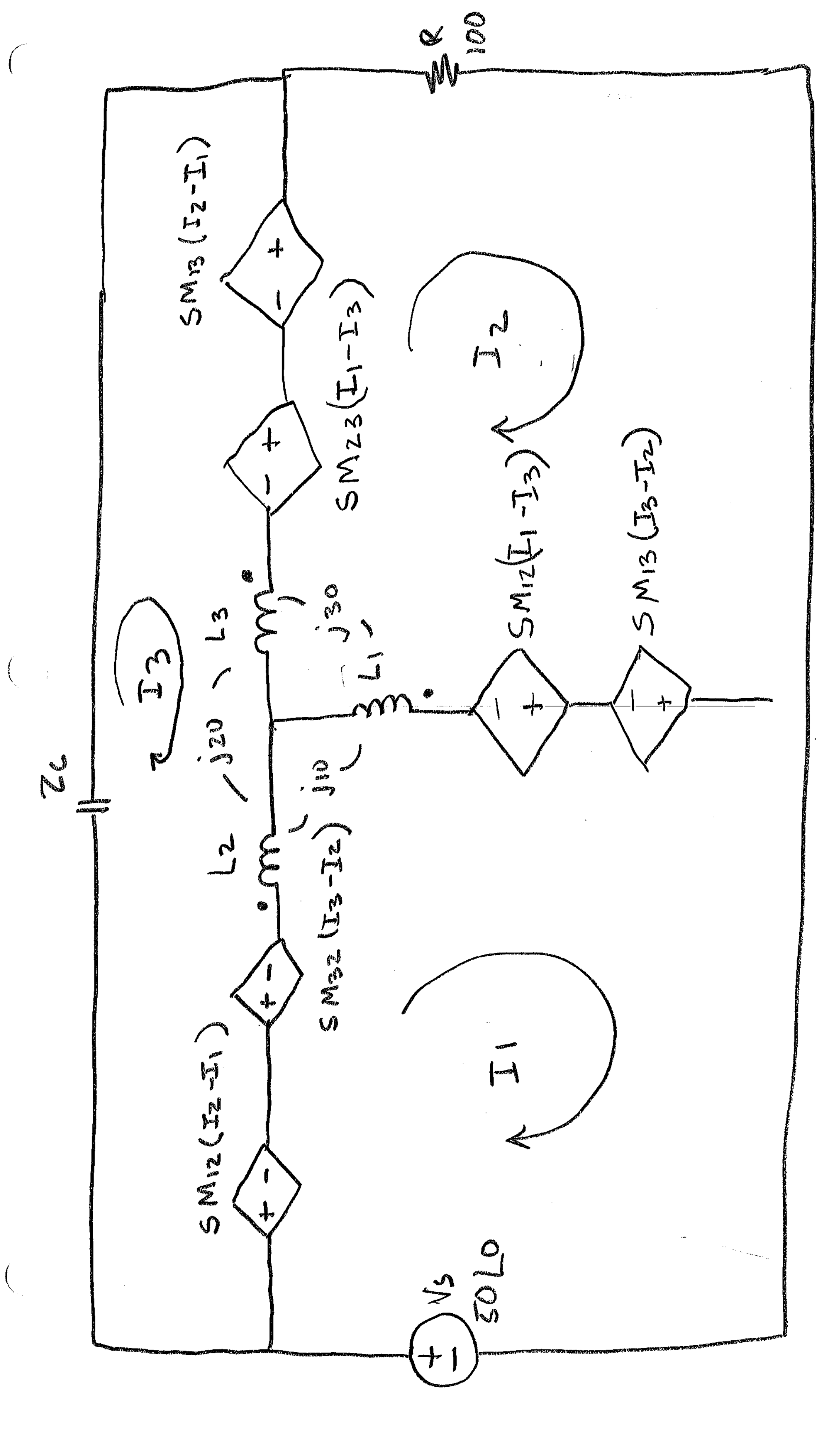
 $SM_{23}(I_1 - I_3)$

(F) $I_{L1} = I_2 - I_1$, $\quad$ I_2 makes dot of L_3 more pos

 $SM_{13}(I_2 - I_1)$

Professor Dorr's secret notes

C



Professor Dorr's Secret notes

1D

(1)

$$-Z_{12}I_3$$

$$-V_S + Z_{12}(I_2-I_1) + Z_{32}(I_3-I_2) + Z_{L2}(I_1 - I_3) + Z_{L1}(I_1-I_2) - Z_{12}I_1 - Z_{13}(I_3-I_2) = 0$$

$$-Z_{L2} \quad -Z_{12}$$

$$I_1(-Z_{12}+Z_{L2}+Z_{L1}-Z_{12}) + I_2(Z_{12}-Z_{32}-Z_{L1}+Z_{13}) + I_3(Z_{32}-Z_{12}) = V_S$$

(2)

$$-Z_{12}I_3$$

$$Z_{13}(I_3-I_2) + Z_{12}I_1 + Z_{L1}(I_2-I_1) + Z_{L3}(I_2-I_3) - Z_{23}I_1 - Z_{13}(I_2-I_1) + I_2R = 0$$

$$+Z_{23}I_3$$

$$I_1(Z_{12}-Z_{L1}-Z_{23}+Z_{13}) + I_2(-Z_{13}+Z_{L1}+Z_{L3}-Z_{13}+R)$$

$$-Z_{12}+Z_{23}$$

$$+ I_3(Z_{13}-Z_{L3}) = 0$$

(3)

$$(I_3-I_1)Z_{L2} - Z_{32}(I_3-I_2) - Z_{12}(I_2-I_1) + I_3Z_C$$

$$-Z_{23}I_3$$

$$+ Z_{13}(I_2-I_1) + Z_{23}I_1 + (I_3-I_2)Z_{L3} = 0$$

$$I_1(-Z_{L2}+Z_{12}-Z_{13}+Z_{23}) + I_2(Z_{32}-Z_{12}+Z_{13}-Z_{L3})$$

$$+ I_3(Z_{L2}-Z_{32}+Z_C+Z_{L3}) = 0$$

$$-Z_{23}$$

SOLVING GIVES

$$I_1 = 0.1885 + j0.143$$
$$I_2 = -0.0808 + j0.2962$$
$$I_3 = 0.5923 + j1.1615 = I_0 = 1.30\angle 62.98^\circ$$