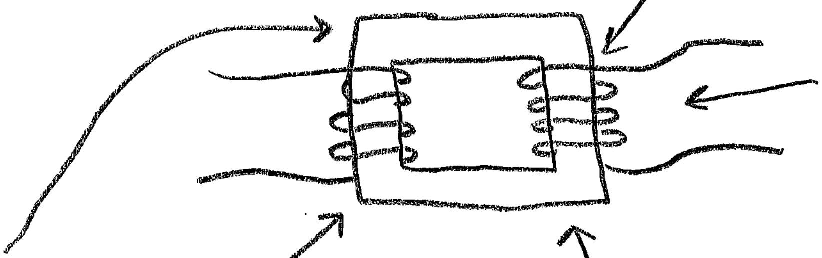
LECTURE 14 - IDEAL TRANSFORMERS

ideal transformer



$$\begin{aligned} ① \quad V_1 &= j\omega L_1 I_1 + j\omega M I_2 \\ ② \quad V_2 &= j\omega M I_1 + j\omega L_2 I_2 \end{aligned} \Bigg\} \text{ Transformer phasor Equations } 13.49a,\ 13.49b$$

Substituting ① into ② gives (solve ① for I_1)

$$② \qquad V_2 = j\omega L_2 I_2 + \frac{M V_1}{L_1} - \frac{j\omega M^2 I_2}{L_1}$$

and since $k = 1$, $M = \sqrt{L_1 L_2}$ so

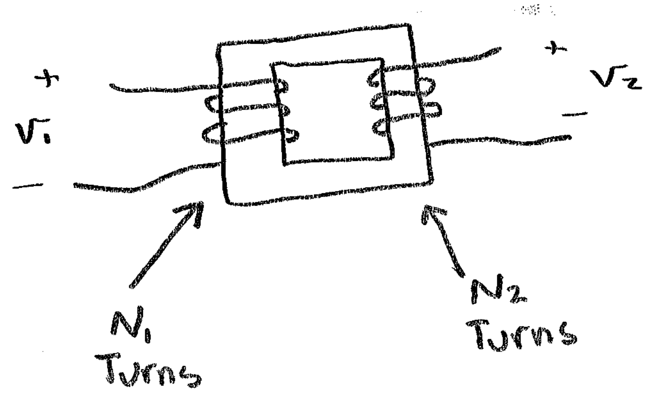
$$V_2 = j\omega L_2 I_2 + \frac{\sqrt{L_1 L_2}\, V_1}{L_1} - \frac{j\omega L_1 L_2 I_2}{L_1}$$

these cancel

$$V_2 = \frac{\sqrt{L_1 L_2}}{L_1} V_1 = \sqrt{\frac{L_2}{L_1}}\, V_1 = n V_1$$

where $n = \sqrt{\frac{L_2}{L_1}}$ = Turns ratio $= \frac{N_2}{N_1}$

We can also show this by



$$V_1(t) = N_1 \frac{d\phi_1}{dt} \quad , \quad V_2(t) = N_2 \frac{d\phi_2}{dt}$$

$K = 1$ so

$$\frac{d\phi_1}{dt} = \frac{d\phi_2}{dt} = \frac{d\phi}{dt}$$

and

$$\frac{V_2(t)}{V_1(t)} = \frac{N_2 \frac{d\phi}{dt}}{N_1 \frac{d\phi}{dt}} = \frac{N_2}{N_1} = n$$

Symbol for ideal transformer

Ideal transformer absorbs no power so

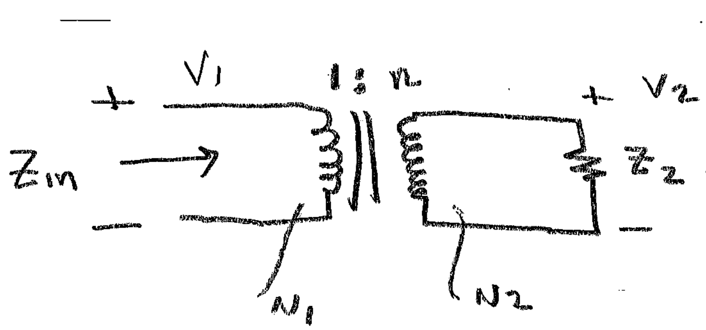
$$v_1(t) \cdot i_1(t) = v_2(t) \cdot i_2(t) \qquad \text{EQ } 13.53$$

in phasor form

$$V_1 I_1 = V_2 I_2$$

or
$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = n \qquad \text{EQ } 13.55$$

We can develop a formula for reflected impedance

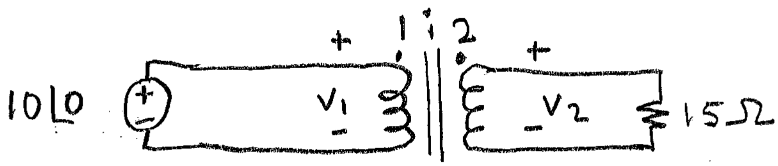


Since $V_2 = nV_1$ and $I_2 = \dfrac{I_1}{n}$

$$V_1 = \frac{V_2}{n} \qquad I_1 = nI_2$$

$$\frac{V_1}{I_1} = Z_{in} = \frac{V_2/n}{nI_2} = \frac{V_2}{I_2} \times \frac{1}{n^2} = \frac{1}{n^2} Z_2$$

Sanity check:



$n = \frac{N_2}{N_1} = 2$, $\frac{V_2}{V_1} = 2$ so $V_2 = 2 \times 10 = 20\angle 0$ V

So $I_2 = \frac{20V}{15\,\Omega} = 1.33\,A$, $P_2 = \frac{1}{2} I_2^2 \times 15 = 13.33\,W$

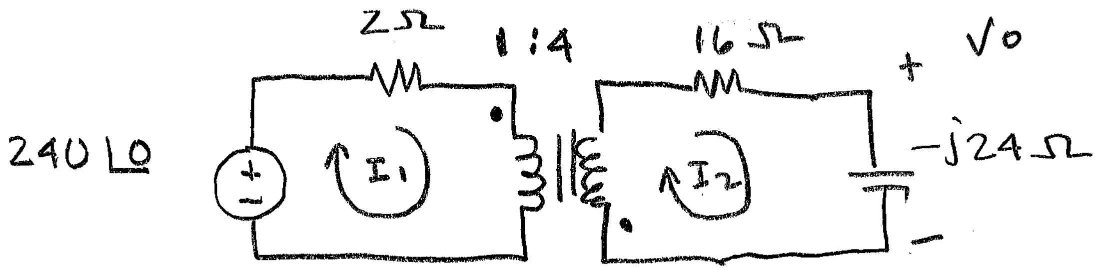
$$Z_{pri} = \frac{Z_L}{n^2} = \frac{15}{2^2} = 3.75\,\Omega$$

$I_1 = \frac{10}{3.75} = 2.67\,A$ $\qquad \underbrace{\frac{I_2}{I_1} = \frac{1.33}{2.67} = \frac{1}{2} = \frac{1}{n}}_{\text{Checks}}$

$P_1 = \frac{1}{2}\frac{V_s^2}{Z_{pri}} = \frac{1}{2} \times \frac{10^2}{3.75} = 13.33W$ checks

PRACTICE PROBLEM 13.8 – Pg 578

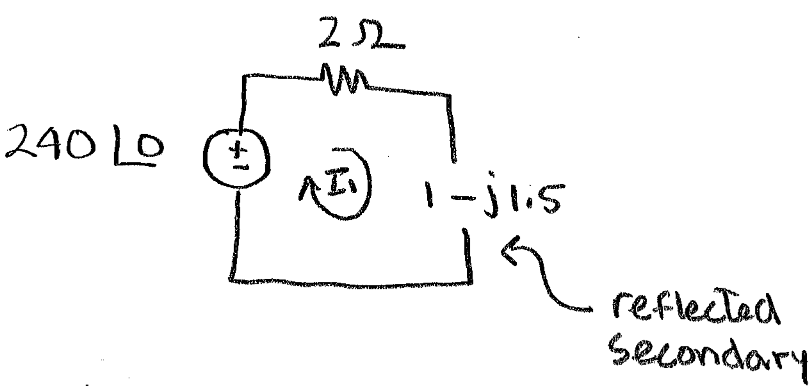
Find V_o



STRATEGY 1

1) Reflect load to transformer input
2) Compute current I_1 — note dots
3) Use relationship $\frac{I_2}{I_1} = \frac{-1}{n}$ to get I_2
4) $V_o = I_2 Z_c = I_2 \times -j24$

Load is $16 - j24$, reflects to $\frac{16-j24}{n^2} = \frac{16-j24}{4^2} = 1 - j1.5\ \Omega$



$$I_1 = \frac{240\angle 0}{1 - j1.5 + 2} = 64 + j32\ A$$

Now reflect I_1 to I_2, $I_2 = -I_1 \times \frac{1}{n} = -(64 + j32) \times \frac{1}{4}$

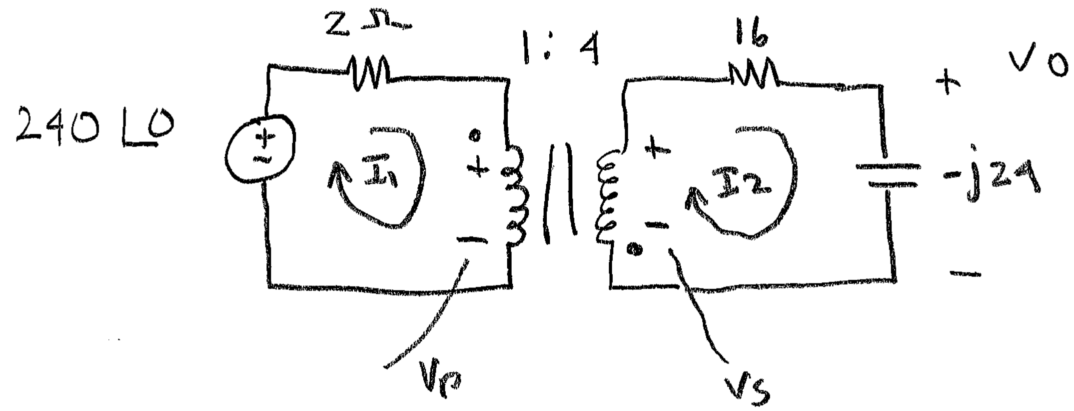
$$I_2 = -16 - j8$$

and $V_o = I_2 \times Z_c = -(16 + j8) \times -j24 = \underline{429.33\angle 116.56\ V}$

PRACTICE PROBLEM 13.8 cont.

Strategy 2

1) solve for mesh currents I_1, I_2
2) $V_o = I_2 Z_C$



① $-V_{in} + 2I_1 + V_p = 0$

② $-V_s + 16 I_2 - j24 I_2 = 0$

} 2 EQ, UNKNOWNS, V_p, V_s, I_1, I_2

Note that → watch that dot!

$$V_p = -\frac{V_s}{n} \quad \text{and} \quad I_1 = -n I_2$$

So

① $-V_{in} - 2nI_2 - \frac{V_s}{n} = 0 \longrightarrow \underbrace{-2n I_2 - \frac{1}{n} V_s}_{\text{UNKNOWNS}} = V_{in}$

② $-V_s + 16 I_2 - j24 I_2 = 0 \longrightarrow (16 - j24) I_2 - V_s = 0$

So

$$\begin{bmatrix} I_2 \\ V_s \end{bmatrix} = \begin{bmatrix} -2n & -1/n \\ 16-j24 & -1 \end{bmatrix}^{-1} \begin{bmatrix} V_{in} \\ 0 \end{bmatrix}$$

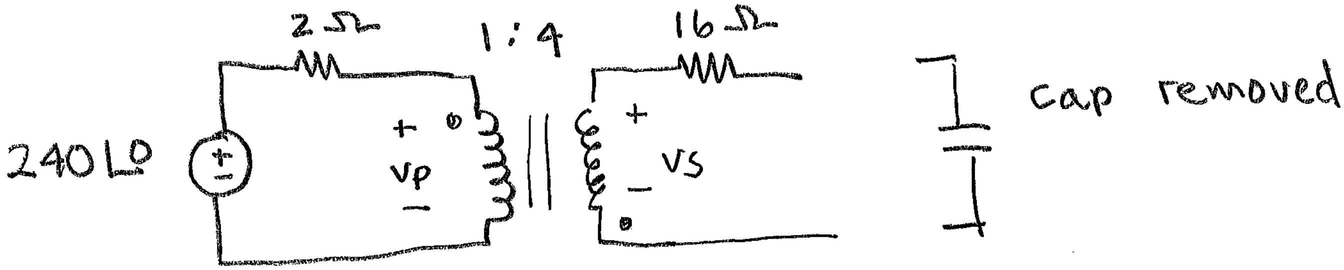
and $\underbrace{I_2 = -16 - j8}_{\text{Just like before.}}$

So $V_o = I_C \times Z_C = -(16 + j8) \times -j24 = 429.33 \angle 116.56°$

Practice problem 13.8 Strategy #3

1) Represent everything to the left of the capacitor as a Thevenin source

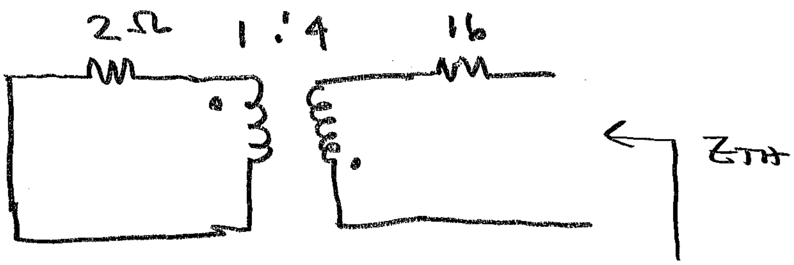
2) Load circuit with capacitor and use voltage divider



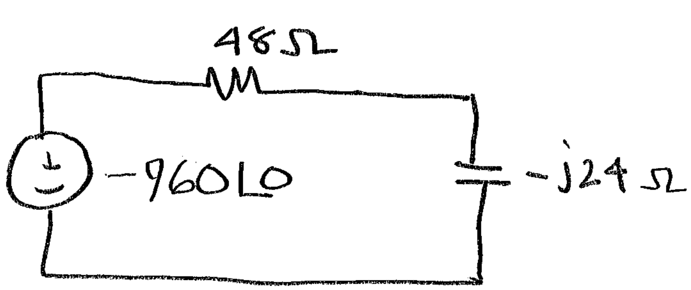
No current in primary or secondary so $V_p = 240V$

$$V_S = -n \times V_p = -4 \times 240 = -960V = V_{TH}.$$

Get Z_{TH} by shorting voltage source and measuring impedance looking into Thevenin source



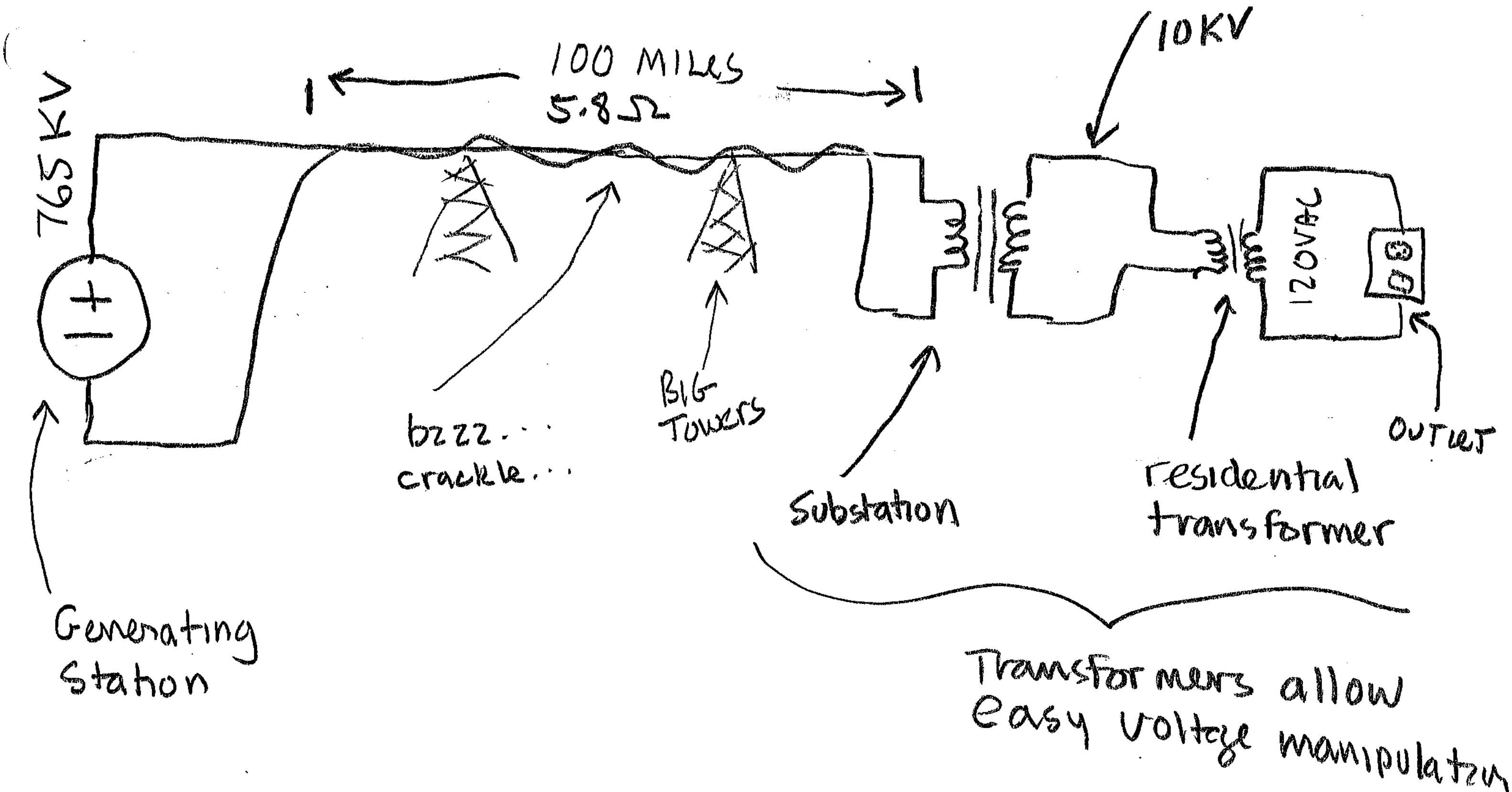
$$Z_{TH} = 16 + 2 \times n^2 = 16 + 2 \times 4^2 = 48\Omega$$



$$V_0 = -960 \times \frac{-j24}{-j24+48}$$

$$= 429.33\angle 116.56^\circ$$

EXAMPLE - ELECTRIC POWER DISSIPATION



Substation transformer $\frac{V_2}{V_1} = \frac{10\,KV}{765\,KV} = \frac{N_2}{N_1} = \frac{1}{76.5}$

residential transformer $\frac{V_2}{V_1} = \frac{120}{10{,}000} = \frac{1}{83}$

Our system Feeds half of San Diego,
Say 316,000 homes at 3.16 KW apiece.
Thats 1×10^9 W or a GW

Distance from substation to consumer is insignificant compared to 100 mile transmission distance, so all losses are on 765 KV Line.

$$\text{Current (765KV Line)} = \frac{P}{V} = \frac{1\text{ GW}}{765\text{ KV}} = \underline{1307\text{ A}}$$

so loss in 765 KV Line $= I^2R = 1307^2 \times 5.8 = \underline{9.9\text{ MW}}$

and $$\frac{\text{Pwr Lost}}{\text{Pwr Generated}} = \frac{9.9\text{ MW}}{1\text{ GW}} \times 100 \simeq 1\%$$

Say we distributed at 100 KV, then

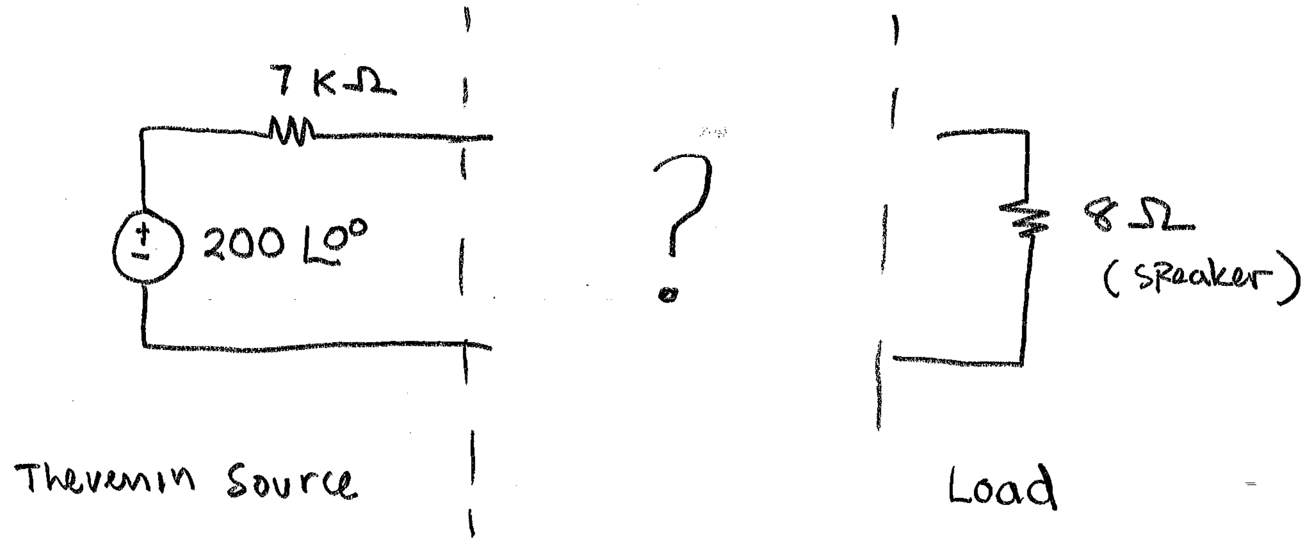
$$I = \frac{1\text{ GW}}{100\text{ KV}} = 10{,}000\text{ A}$$

Line Loss $= I^2R = 10{,}000^2 \times 5.88 = 588$ MW

and $$\frac{\text{PWR Lost}}{\text{PWR generated}} = \frac{588\text{ MW}}{1\text{ GW}} = 59\%$$

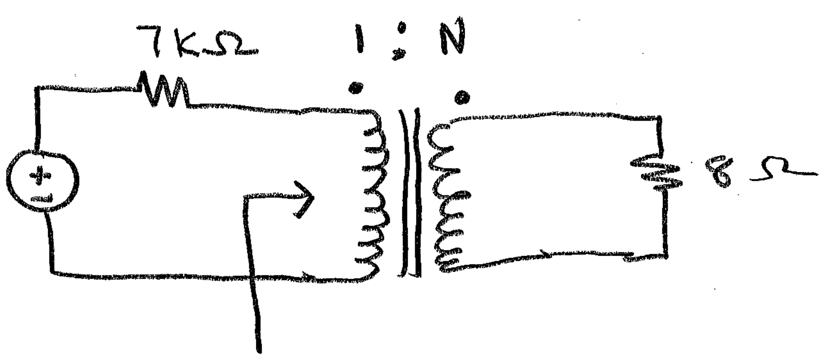
Towers are expensive. This is why they're used

EXAMPLE – Maximum Power Transfer



In this problem we are given a source impedance and a load impedance and we need to design the "Matching Network"

Say we connect them with a transformer.



$$Z_{in} = \frac{8\,\Omega}{N^2}$$

To match we want $Z_{in} = 7\,K\Omega$ so

$$7\,K\Omega = \frac{8\,\Omega}{N^2} \quad \text{or} \quad N = 0.0338 = \frac{1}{29.5}$$