LECTURE 13 - MUTUAL INDUCTANCE

- Conductively coupled circuits
- Magnetically coupled circuits

Why?

- Electrical isolation
- Step up/down for voltages and currents
- Impedance matching for maximum power transfer
- Filters!!

Introduction

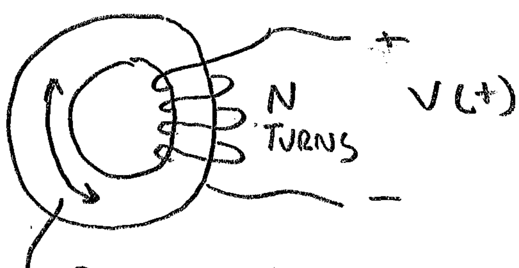
Electricity and magnetism are related by Maxwell's equations.

For our purposes we simplify to Faraday's Law:

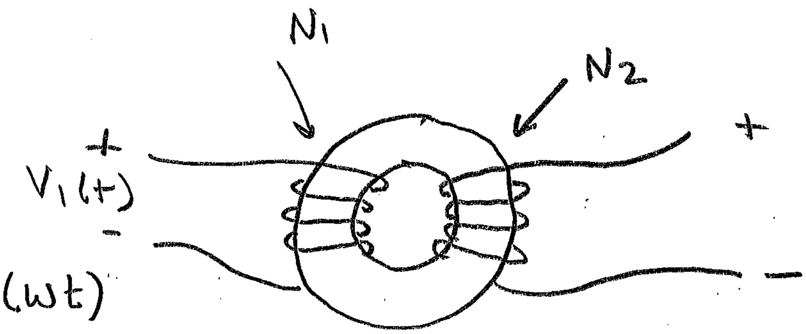
$$V(t) = N \frac{d\phi}{dt}$$ change in flux through a surface.

Flux can induce voltage.

Voltage can induce flux.



flux, Ø, is in core.



$$V_2 = \frac{N_2}{N_1} \sin(\omega t)$$

$$V_1(t) = \sin(\omega t)$$

$$\phi(t) = \frac{1}{N_1} \int \sin(\omega t)\, dt$$

$$V_2 = N_2 \frac{d\phi}{dt}$$

$$= \frac{N_2}{N_1} \sin(\omega t)$$

- Ideal transformer will be introduced in Sec 13.5
- Not all of the flux is coupled by both coils, some leaks out

we start with

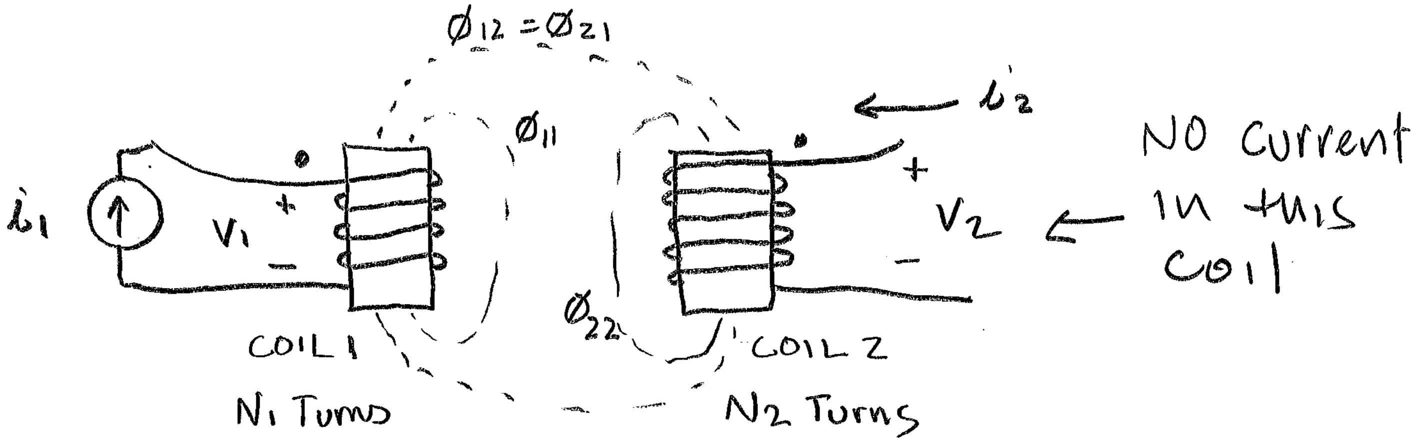
$$V(t) = N \frac{d\phi}{dt}$$

But, flux is produced by current

$\frac{d\phi}{dt}$ is flux change with a single turn

so we have

$$V(t) = N \frac{d\phi}{dt} = \underbrace{N \frac{d\phi}{di}}_{\text{INDUCTANCE}} \cdot \frac{di}{dt} = L \frac{di}{dt}$$



Flux from coil 1 $= \phi_1 = \phi_{11} + \phi_{12}$

$$V_1 = N_1 \frac{d\phi_1}{dt} = N_1 \frac{d\phi_1}{di_1} \cdot \frac{di_1}{dt} = L_1 \frac{di_1}{dt} \qquad \text{EQ 13.8}$$

L_1: Self inductance of coil 1

$$V_2 = N_2 \frac{d\phi_{12}}{dt} = N_2 \frac{d\phi_{12}}{di_1} \frac{di_1}{dt} = M_{21} \frac{di_1}{dt} \qquad \text{EQ 13.11}$$

M_{21}: Mutual inductance

We could place current source on the right side and get

$$V_2 = L_2 \frac{di_2}{dt} \qquad \text{EQ } 13.13$$

and

$$V_1 = M_{12} \frac{di_2}{dt} \qquad \text{EQ } 13{,}15$$

My Favorite equations:

$$V_1 = L_1 \frac{di_1}{dt} + M_{21} \frac{di_2}{dt}$$

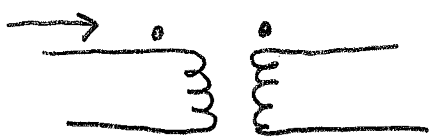
$$V_2 = L_2 \frac{di_2}{dt} + M_{12} \frac{di_1}{dt}$$

$M_{12} = M_{21}$

So "Mutual inductance is the ability of one inductor to induce a voltage across a neighboring inductor measured in Henries"

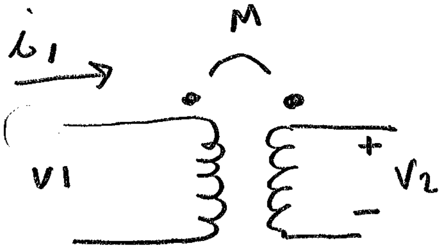
Bottom of Pg 538

<u>DOts</u> – Connecting the physical part with the math

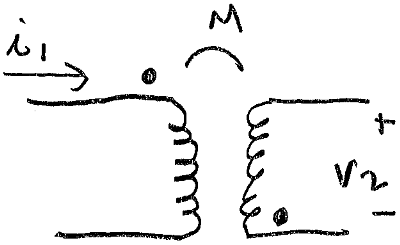


"If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil"

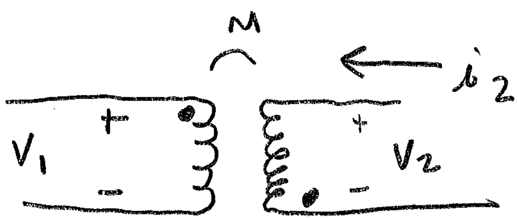
Pg 539



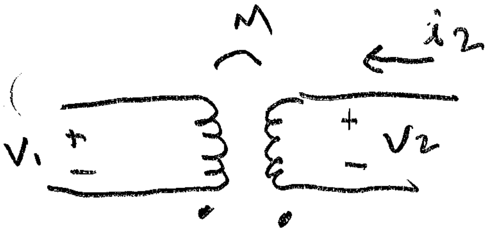
$v_2 = M \frac{di_1}{dt}$



$v_2 = -M \frac{di_1}{dt}$



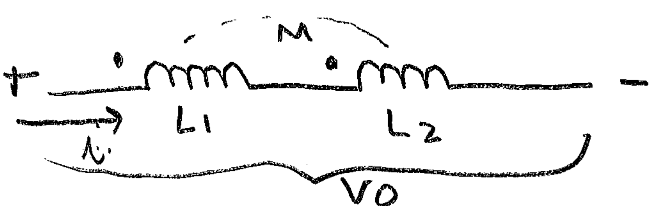
$v_1 = -M \frac{di_2}{dt}$



$v_1 = M \frac{di_2}{dt}$

Fig 13.5 in Text

Coupled coils in series

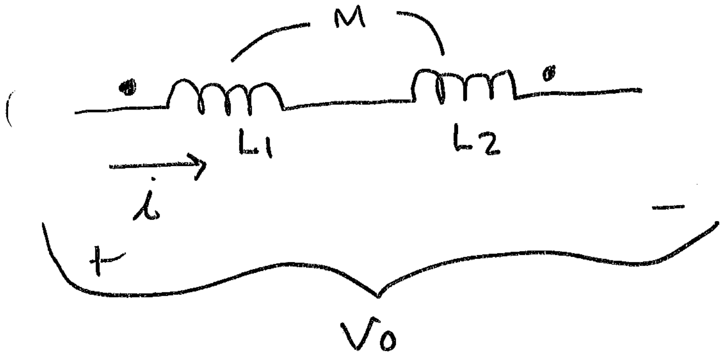


If not coupled, $L_s = L_1 + L_2$

If coupled, use KVL

$$V_0 = L_1 \frac{di}{dt} + \underbrace{M\frac{di}{dt}}_{\text{induced in coil 2}} + L_2 \frac{di}{dt} + \underbrace{M\frac{di}{dt}}_{\text{induced in coil 1}}$$

$V_0 = (L_1 + L_2 + 2M)\, di/dt$ so $L_s = L_1 + L_2 + 2M \rightarrow$ EQ 13.8



$$V_0 = L_1\frac{di}{dt} - M\frac{di_1}{dt} + L_2\frac{di}{dt} - M\frac{di}{dt}$$

$$V_0 = (L_1 + L_2 - 2M)\,di/dt$$

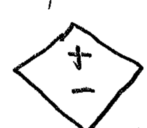
so $L_s = L_1 + L_2 - 2M$ EQ 13.19

Housekeeping to make problems easier – pg 561

$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = j\omega L_1 I_1 + j\omega M I_2$$

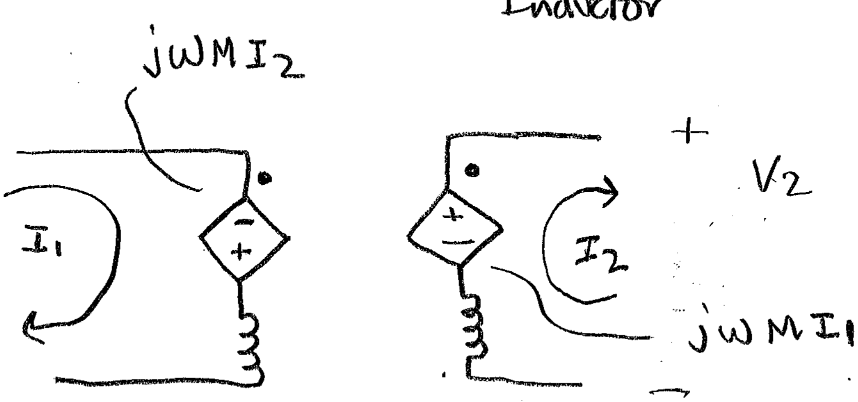
$$V_2 = \underbrace{L_2\frac{di_2}{dt} + M\frac{di_1}{dt}}_{\text{differential}} = \underbrace{j\omega L_2 I_2 + j\omega M I_1}_{\text{Phasor}}$$

Regular Inductor



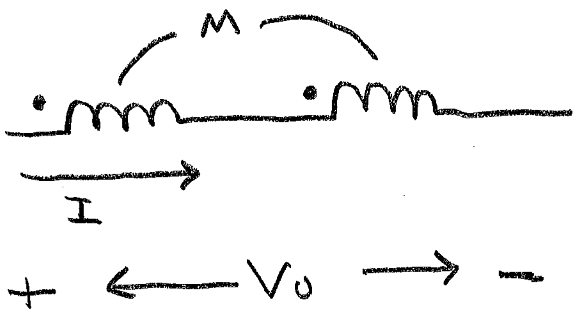
$j\omega M I_2$

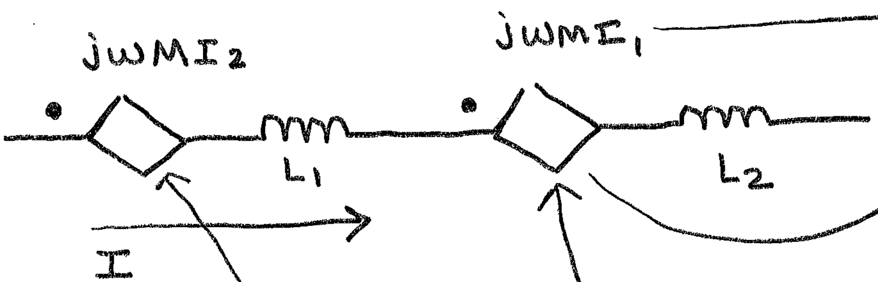
dependent source



REVISIT THE SERIES COIL PROBLEM USING PHASORS

Find inductance of series combination



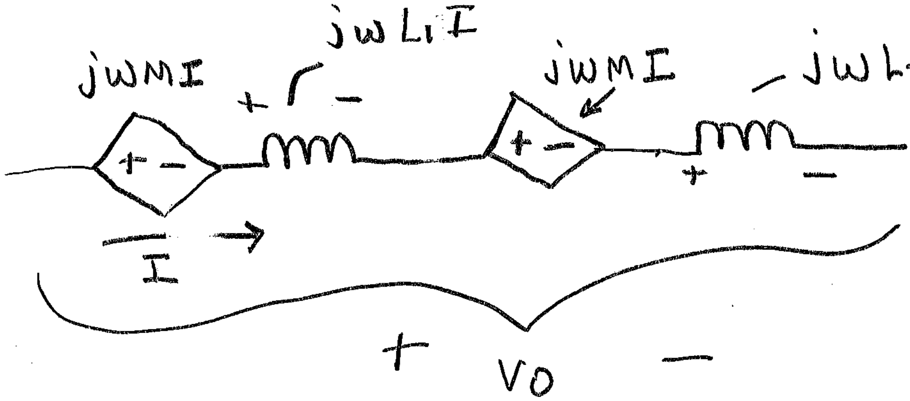


know value but not polarity.

I draw the source between coil and its dot

Current enters dot of L_2 so it makes dot of L_1 more positive

Current enters dot of L_1 so it makes the dot of L_2 more positive



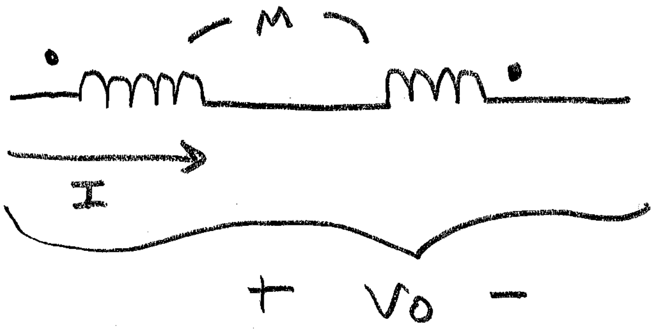
No more dots!

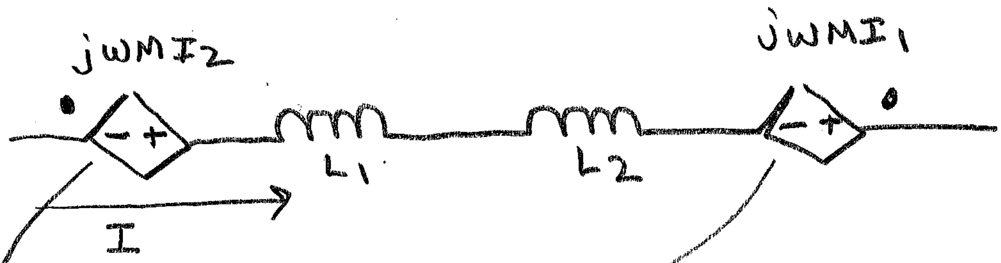
Add voltages starting at negative end.

$$V_0 = j\omega L_2 I + j\omega M I + j\omega L I + j\omega M I$$

$$= j\omega I \underbrace{(L_1 + L_2 + 2M)}_{\text{Equivalent inductance}}$$

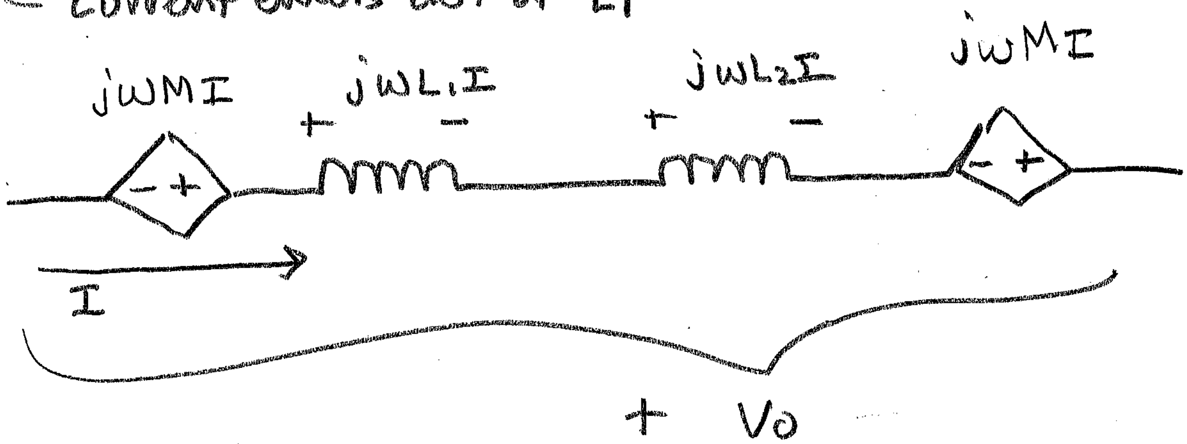
Now Reverse dot on one coil and re-compute inductance.





current enters non dot of L2

current enters dot of L1

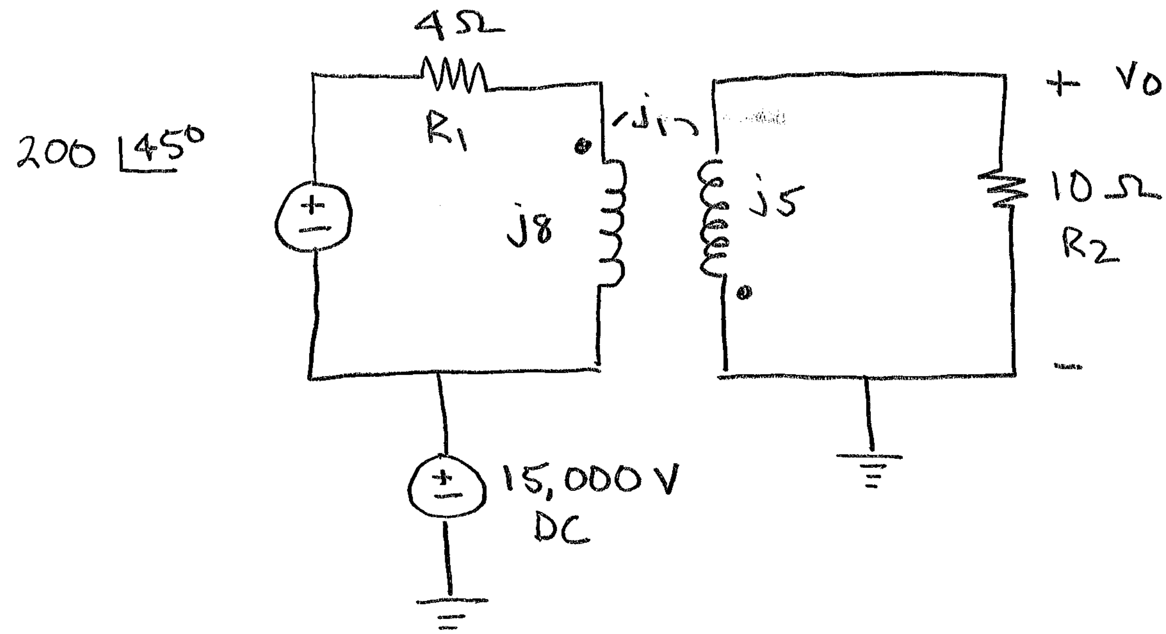


Add voltages starting at negative end.

$$V_o = -j\omega MI + j\omega L_2 I + j\omega L_1 I - j\omega MI$$

$$V_o = j\omega I\underbrace{(-M + L_2 + L_1 - M)}_{\text{EQUIV INDUCTANCE}}$$

PRACTICE Problem 13.1 Pg 562



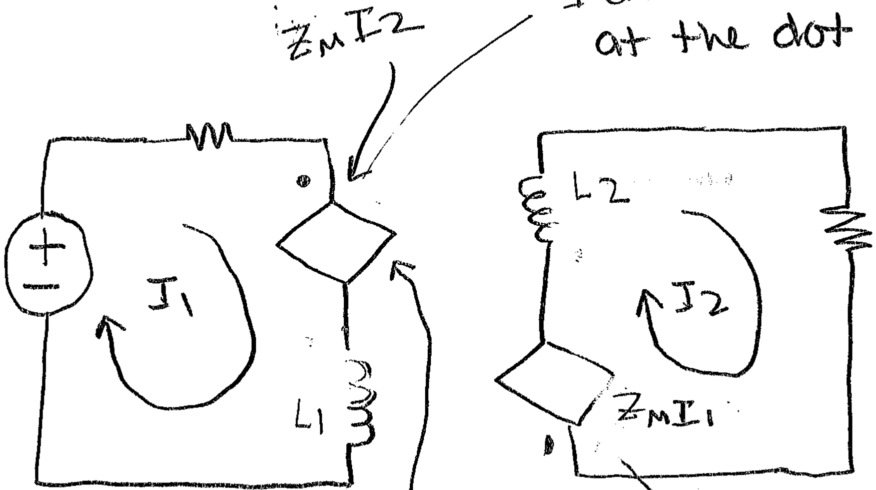
Find Vo

Strategy

1) Deal with the 15,000 Volt source
2) Redraw with dependent sources
3) Use dots to establish polarity of dependent sources
4) Forget about the dots
5) Solve for I_1, I_2
6) $V_o = I_2 \cdot 10\Omega$

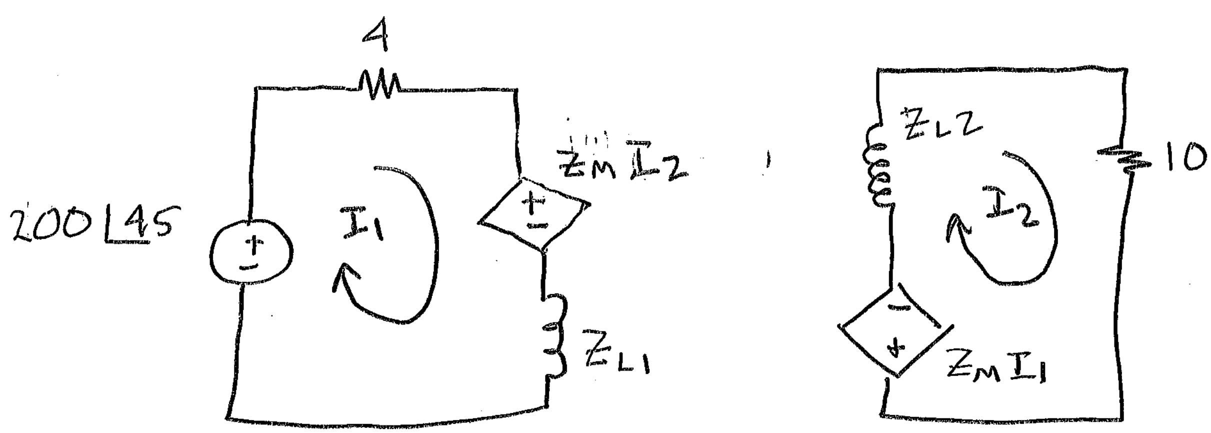
Deal with DC .....

I draw the source at the dot



when I_1 Goes into dot of L_1 it makes dot of L_2 more positive. so 

When I_2 goes into dot of L_2 it makes dot of L_1 more positive so 



(1) $-V_S + 4I_1 + Z_M I_2 + Z_{L1} I_1 = 0$

$$(4 + Z_{L1}) I_1 + Z_M I_2 = V_S$$

(2) $Z_M I_1 + Z_{L2} I_2 + 10 I_2 = 0$

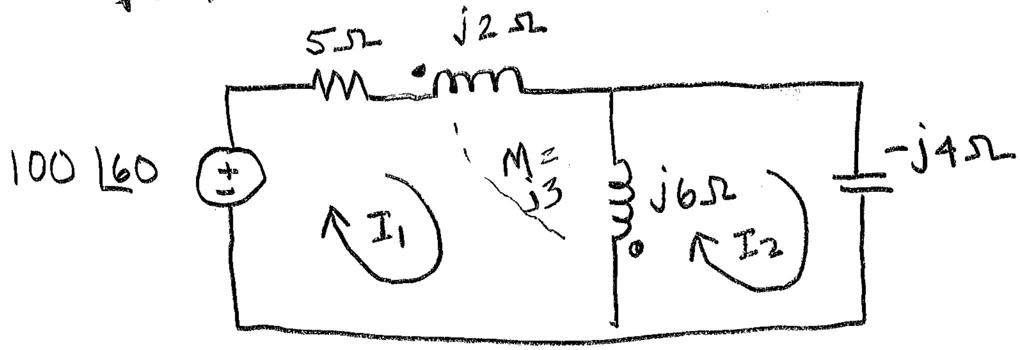
$$Z_M I_1 + (Z_{L2} + 10) I_2 = 0$$

$$\begin{bmatrix} 4 + Z_{L1} & Z_M \\ Z_M & Z_{L2} + 10 \end{bmatrix}^{-1} \begin{bmatrix} V_S \\ 0 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 21.28 - j6.85 \\ -1.4 - j1.42 \end{bmatrix}$$

and $I_2 \times 10 = V_O = (-1.4 - j1.42) \times 10 = 20\angle -134^\circ$

Practice Problem 13.2 Pg 563

Find Mesh Currents

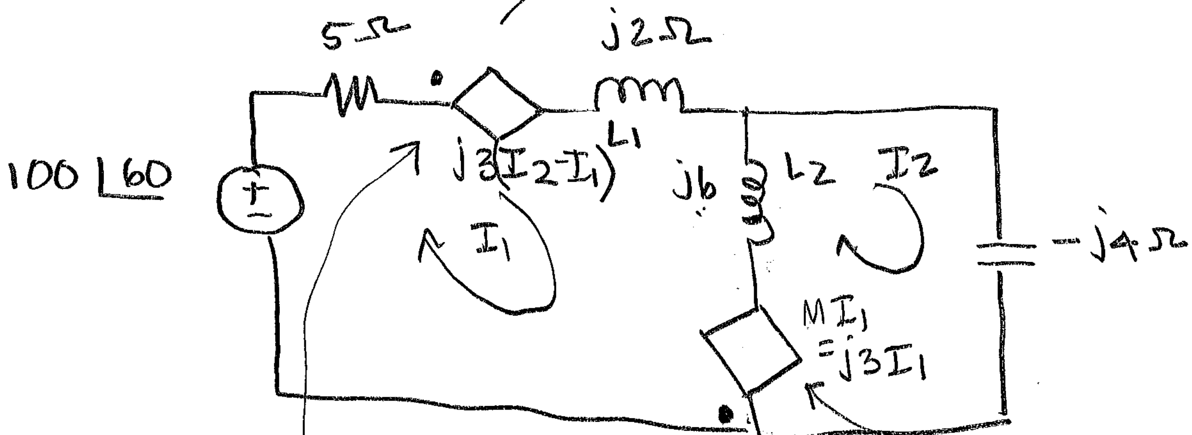


Practical Problem?

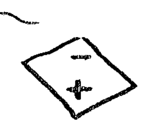
STRATEGY

1) Redraw with dependent sources
2) Use dots to establish polarity of dependent sources.
3) Forget about the dots
4) Write mesh equations
5) Get currents

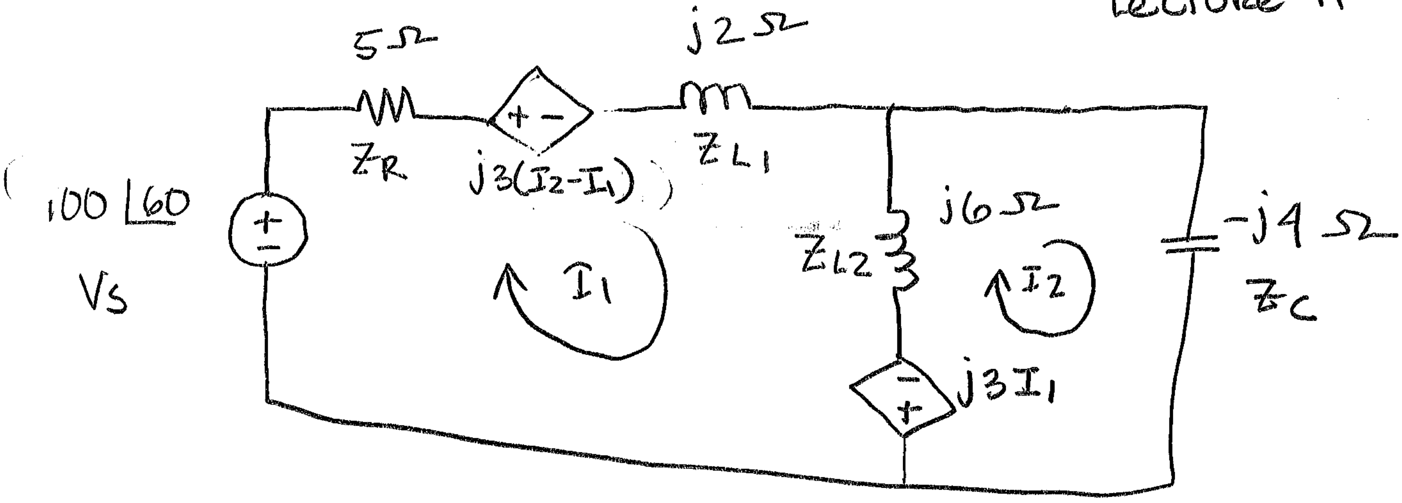
I always draw the voltage source next to the dot



I_1 makes dot of L_2 more positive so



I_2 makes dot of L_1 more positive so



No more dots

(1) $-V_S + I_1 Z_R + Z_M(I_2 - I_1) + I_1 Z_{L1} + (I_1 - I_2) Z_{L2} - I_1 Z_M = 0$

$$(Z_R + Z_{L1} + Z_{L2} - 2Z_M) I_1 + (Z_M - Z_{L2}) I_2 = V_S$$

(2) $Z_M I_1 + (I_2 - I_1) Z_{L2} + I_2 Z_C = 0$

$$(Z_M - Z_{L2}) I_1 + (Z_{L2} + Z_C) I_2 = 0$$

$$\begin{bmatrix} Z_R + Z_{L1} + Z_{L2} - 2Z_M & Z_M - Z_{L2} \\ Z_M - Z_{L2} & Z_{L2} + Z_C \end{bmatrix}^{-1} \begin{bmatrix} 100\angle{-60} \\ 0 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$I_1 = 1.07 + j17.86 = 17.89\angle 86.56$

$I_2 = 1.61 + j26.78 = 26.83\angle 86.56$

SECTION 13.3 - Energy in a Coupled Circuit

pg 564 & 565 derive;

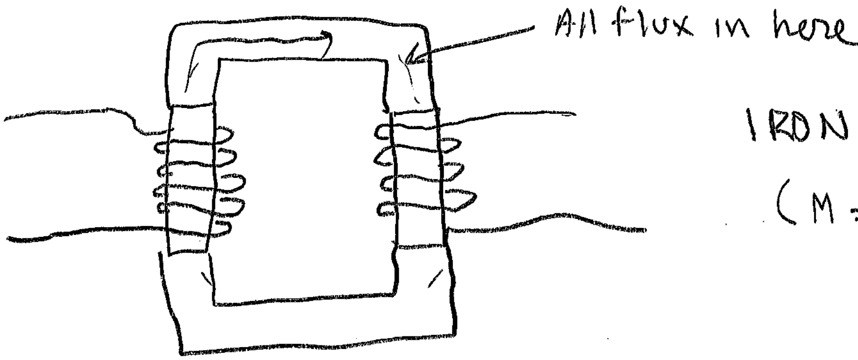
$M_{12} = M_{21} = M$ EQ 13.30a

$M \leq \sqrt{L_1 L_2}$ EQ 13.35 with perfect coupling

K = Coefficent of coupling

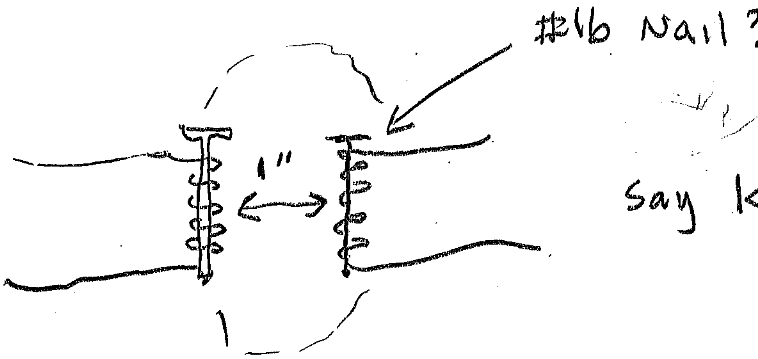
$$K = \frac{M}{\sqrt{L_1 L_2}}$$ EQ 13.36

so $0 \leq K \leq 1$ or $0 \leq M \leq \sqrt{L_1 L_2}$

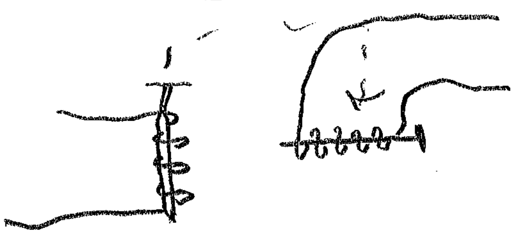


IRON CORE $K \simeq 1$

$(M = \sqrt{L_1 L_2})$



Say $K \simeq 0.5$



Fields orthogonal
no coupling $K = 0$