EXAM 1 REVIEW - EE-310

LECTURE #2

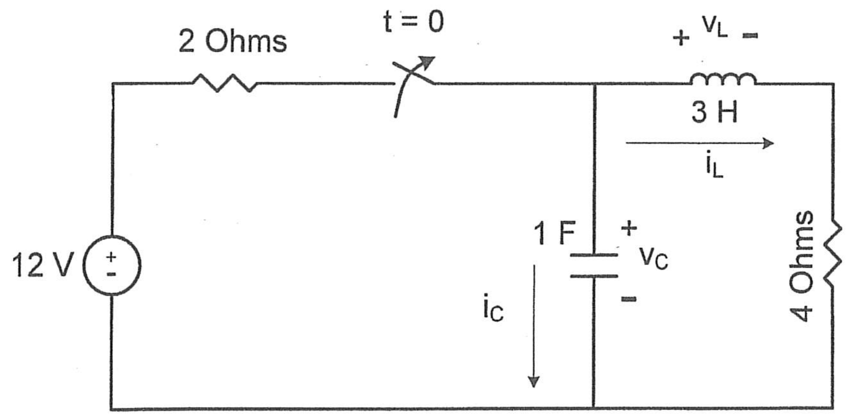
The Rules

"Verbal"	Equation	
Inductor current cannot change instantaneously"	$i_L(0^+) = i_L(0^-)$	1
Capacitor voltage cannot change instantaneously	$V_C(0^+) = V_C(0^-)$	2
Inductor voltage can change instantaneously	$V_L = L\frac{di}{dt}$	3
Capacitor current can change instantaneously	$i_C = C\frac{dv}{dt}$	4
At steady state $\frac{di}{dt} = 0$ so inductor voltage is zero	$V_L(\infty) = 0$	5
At steady state $\frac{dv}{dt} = 0$ so $i_C = 0$	$I_C(\infty) = 0$	6

EE310 **Midterm Exam #1**

Name: DORR Red ID: ____________

1) After being closed a long time, the switch opens at t=0. Find: (a) $v_c(0^+)$, (b) $i_c(0^+)$, (c) $dv_c/dt(0^+)$, (d) $v_L(0^+)$, (e) $i_L(0^+)$, (f) $dv_L/dt(0^+)$. (20%)



(3) (a) $V_c(0^+) = \frac{4}{4+2} \times 12 = 8\,V$

(4) (b) $i_L(0^+) = i_L(0^-) = \frac{12}{4+2} = 2A. \quad i_L = -i_C = -2A$

(4) (c) $i_c = C\frac{dv}{dt}, \quad \frac{dv_c}{dt} = \frac{i}{C} = \frac{-2}{1} = -2\,V/s$

(3) (d) Use KVL $\quad -V_c(0^+) + V_L(0^+) + 4\,i_L(0^+) = 0$

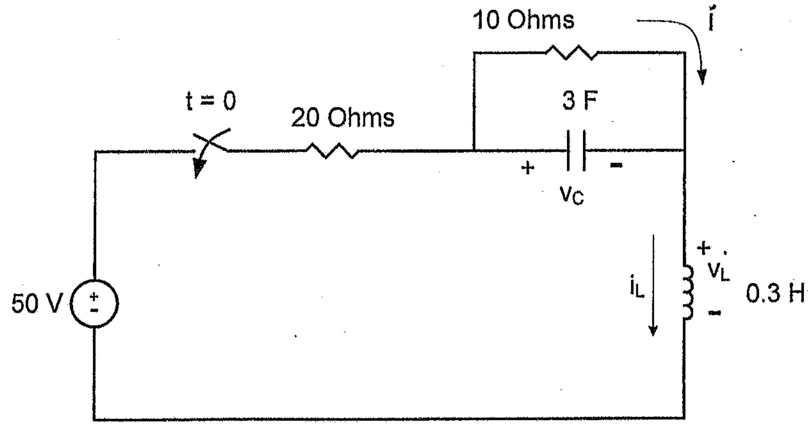
$V_L(0^+) = V_c(0^+) - 4\,i_L(0^+) = 8 - 4\cdot(2) = 0\,V$

(3) (e) $i_L(0^+) = i_L(0^-) = 2A$ (From part b)

(3) (f) Since $V_L(0^+) = 0 = L\cdot\frac{di_L}{dt}, \quad \frac{di_L}{dt} = \frac{di_R}{dt} = \frac{dV_R}{dt} = 0$

Therefore $\frac{dV_L}{dt} = \frac{dV_c}{dt} = -2\,V/s$

2) After being ~~closed~~ open a long time, the switch closes at t=0. Find: (a) $i(0^+)$, (b) $di/dt(0^+)$, (c) $i(\infty)$, (d) $dv_c/dt(0^+)$, (e) $i_L(0^+)$, (f) $di_L/dt(0^+)$ (g) $i_L(\infty)$, (15%)



(2) (a) inductor current can't change quickly so $i_L(0^+) = 0$
∴ no current in 20Ω, 3F, or 10Ω and $i(0^+) = 0$.

(3) (b) $\dfrac{di(0^+)}{dt} = \dfrac{1}{R}\dfrac{dV_c(0^+)}{dt} = \dfrac{1}{R}\cdot\dfrac{\cancel{i_c(0^+)}^{\,0}}{C} = 0$ A/s

(2) (c) $i_\infty = \dfrac{50}{10+20} = 1.67$A

(2) (d) $\dfrac{dV_c(0^+)}{dt} = \dfrac{i_c(0^+)}{C} = \dfrac{0}{C} = 0$

(2) (e) $i_L(0^+) = 0$A (can't change inductor current quickly)

(2) (f) $\dfrac{di_L(0^+)}{dt} = \dfrac{50V}{0.3} = 166.7$ A/s → No current so all voltage is across inductor

(2) (g) $i_L(\infty) = \dfrac{50}{20+10} = 1.66$A

Generalized responses

over damped

$$r(t) = R_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$r(0^+) = R_s + A_1 + A_2$$

$$dr/dt\,(0^+) = A_1 s_1 + A_2 s_2$$

so

$$\begin{bmatrix} 1 & 1 \\ s_1 & s_2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} r(0^+) - R_s \\ dr/dt(0^+) \end{bmatrix}$$

or

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ s_1 & s_2 \end{bmatrix}^{-1} \begin{bmatrix} r(0^+) - R_s \\ dr/dt(0^+) \end{bmatrix}$$

Critically damped

$$r(t) = R_s + (A_1 + A_2 t)e^{-\alpha t}$$

$$\left.\begin{aligned} r(0^+) &= R_s + A_1 \\ \frac{dr}{dt}(0^+) &= -\alpha A_1 + A_2 \end{aligned}\right\} \quad \text{so } \begin{aligned} A_1 &= r(0^+) - R_s \\ A_2 &= \frac{dr}{dt}(0^+) + \alpha A_1 \end{aligned}$$

Underdamped

$$r(t) = R_s + (A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t))e^{-\alpha t}$$

$$r(0^+) = R_s + A_1$$

$$\frac{dr}{dt} = \frac{d}{dt}\left[R_s + A_1 \cos(\omega_d t)e^{-\alpha t} + A_2 \sin(\omega_d t)e^{-\alpha t}\right]$$

$$= -\alpha A_1 \cos(\omega_d t)e^{-\alpha t} - A_1 \omega_d \sin(\omega_d t)e^{-\alpha t}$$

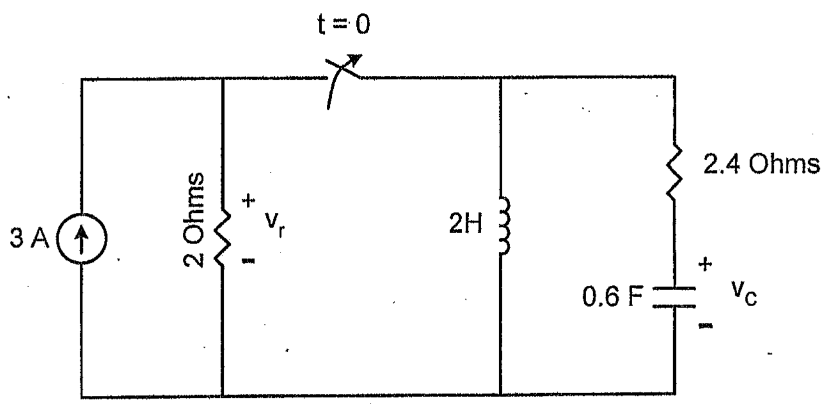
$$+ A_2 \omega_d \cos(\omega_d t)e^{-\alpha t} - \alpha A_2 \sin(\omega_d t)e^{-\alpha t}$$

$$\frac{dr}{dt}(0^+) = -\alpha A_1 + A_2 \omega_d$$

so $A_1 = r(0^+) - R_s$

$$A_2 = \frac{\frac{dr}{dt}(0^+) + \alpha A_1}{\omega_d}$$

3) After being closed for a long time, the switch opens at t= 0. Find: (a) $v_c(0^+)$, (b) $dv_c/dt(0^+)$, (c) $v_c(\infty)$, (d) $v_r(\infty)$, and (e) $v_c(t)$. (20%)



(3) (a) Inductor voltage at 0^- is 0 so $V_c(0^-) = V_c(0^+) = 0$

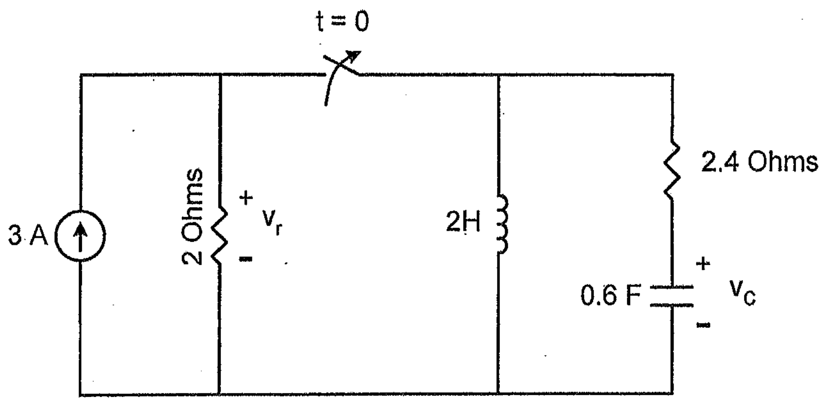
(3) (b) $i_L(0^-) = i_L(0^+) = 3A$, At $t=0^+$ Capacitor must supply current so $i_c(0^+) = -3A$, $\frac{dv_c}{dt}(0^+) = \frac{i_c(0^+)}{C} = \frac{-3}{0.6} = -5V/s$

(3) (c) $V_c(\infty) = 0$. All energy is dissipated

(3) (d) $V_R(\infty) = 3 \times 2 = 6V$

(Work part (e) on the next page)

Problem 3 (cont)



(e)

(1) $\alpha = R/2L = 0.6$

(1) $\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \times 0.6}} = 0.913$

(1) $\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{0.913^2 - 0.6^2} = 0.688 \text{ r/s}$

since $\omega_o > \alpha$ it is underdamped

2 points for General formula $\longrightarrow$ $V_c(t) = e^{-\alpha t}(B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t))$

(1) $B_1 = V_c(0^+) - V_c(\infty) = 0 - 0 = 0$

(1) $B_2 = \dfrac{\frac{dV_c(0^+)}{dt} + \alpha V_c(0^+)}{\omega_d} = \dfrac{-5 - 0}{0.688} = -7.27$

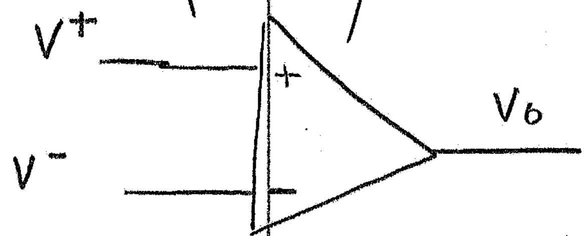
(1) $V_c(t) = -7.27 e^{-0.6t} \sin(0.688t)$

INTRODUCTION to OP-AMPS FOR EE-310

No current flows into inputs V^- and V^+

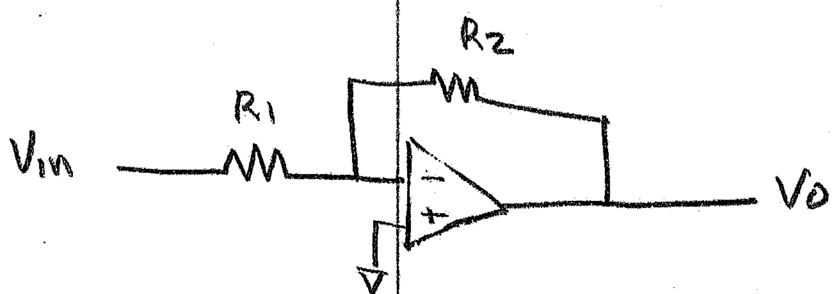
$V_o = \text{Gain} \cdot (V^+ - V^-)$

Gain of op-amp is infinite



IF OUTPUT is between -15 and $+15V$, $V^+ = V^-$

Gain of op amp is infinite. What is the gain of this circuit?



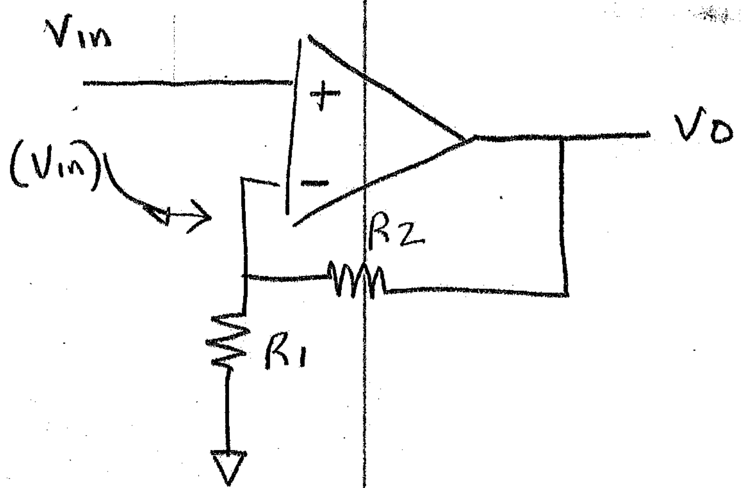
KCL at inverting terminal: (No current into V^-)

Also, since $V^+ = V^-$, $V^- = 0$

$$\frac{V_{in}}{R_1} + \frac{V_o}{R_2} = 0, \quad \frac{V_{in}}{R_1} = -\frac{V_o}{R_2} \quad \text{or Gain} = \frac{V_o}{V_{in}} = -\frac{R_2}{R_1}$$

"Inverting Configuration"

What is the gain of this circuit?



Use KCL again. Remember $V^+ = V^- = V_{in}$

$$\frac{V_{in}}{R_1} = \frac{V_o - V_{in}}{R_2}$$

$$V_{in} R_2 = R_1 (V_o - V_{in}) = R_1 V_o - R_1 V_{in}$$

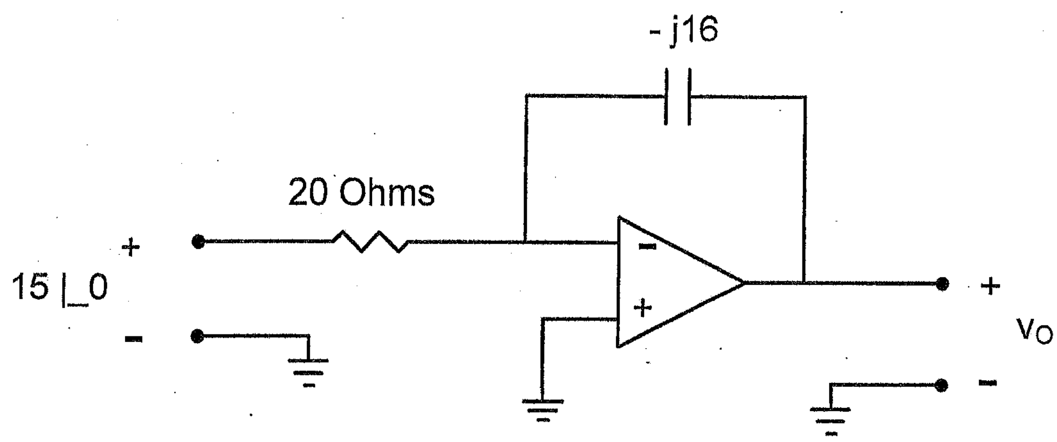
$$V_o R_1 = V_{in} R_2 + V_{in} R_1$$

$$V_o R_1 = V_{in} (R_2 + R_1)$$

or $$\frac{V_o}{V_{in}} = \text{Gain} = \frac{R_2 + R_1}{R_1}$$

"Non-inverting configuration"

6) Determine the output voltage for the circuit below. Express your result as a phasor in polar form. (15%)

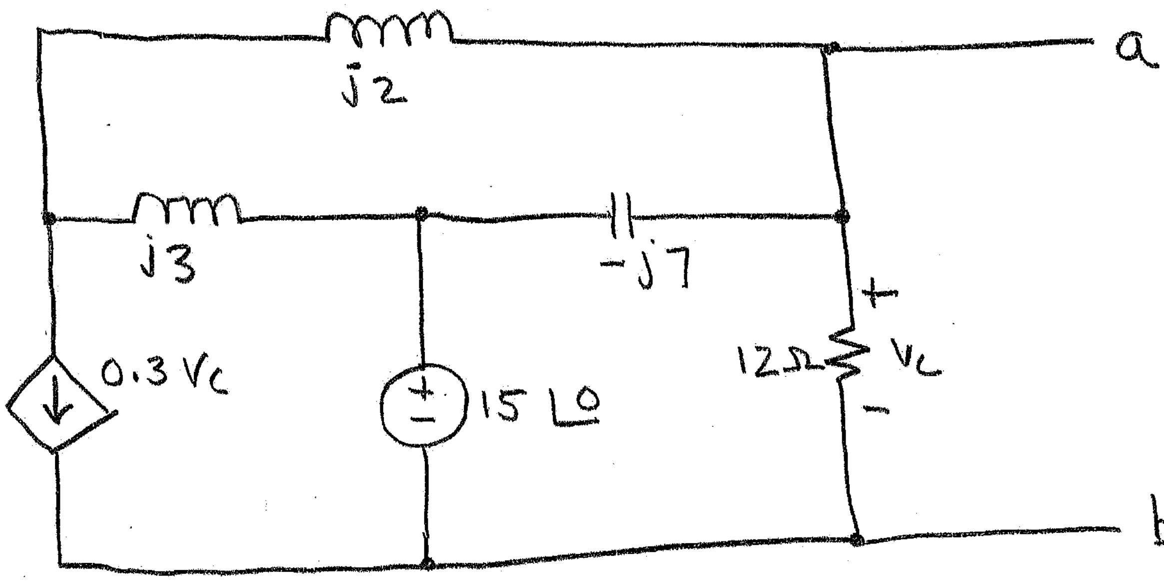


(8) $\frac{V_o}{V_{in}} = -\frac{Z_2}{Z_1}$

(5) $V_o = V_{in} \cdot \left(\frac{-Z_2}{Z_1}\right) = 15\angle 0 \times \frac{-(-j16)}{20} = j12$

(2) $V_o = 12\angle 90$

Supplement for ASSIGN #5



1) Determine the load that can be placed between terminals a and b that will receive maximum power transfer from the circuit.

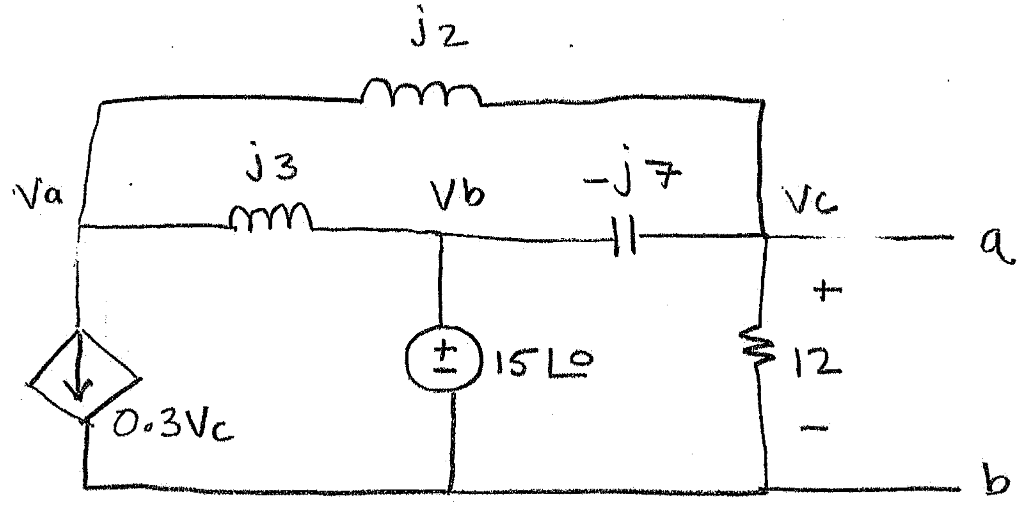
2) Determine the power delivered to this load.

Hints: 1) Use KVL or KCL to get V_{TH}

2) For Thevenin impedance, see "case 2" on pg 140

3) I prefer getting Z_{TH} by computing I_{sc} then

$$Z_{Th} = \frac{V_{TH}}{I_{sc}}$$



Determine the Thevenin equivalent circuit using terminals a and b as the output

Use KCL:

Node voltage Va

$$+0.3\ V_c + \frac{(V_a - V_b)}{j3} + \frac{V_a - V_c}{j2} = 0$$

① $$\left[\frac{1}{j3} + \frac{1}{j2}\right] V_a - \frac{1}{j3} V_b + \left(+0.3 - \frac{1}{j2}\right) V_c = 0$$

Node voltage Vb

↳ it is $15\,V \angle 0$

Node Voltage Vc

② $$\frac{V_c}{12} + \frac{V_c - V_b}{-j7} + \frac{V_c - V_a}{j2} = 0$$

so $$-\frac{1}{j2} V_a + \frac{1}{j7} V_b + \left(\frac{1}{12} + \frac{1}{-j7} + \frac{1}{j2}\right) V_c = 0$$

Substitute $V_b = 15\angle 0$ into ① and ②

① $\left[\frac{1}{j3} + \frac{1}{j2}\right] V_a + \left[+0.3 - \frac{1}{j2}\right] V_c = \frac{15}{j3}$

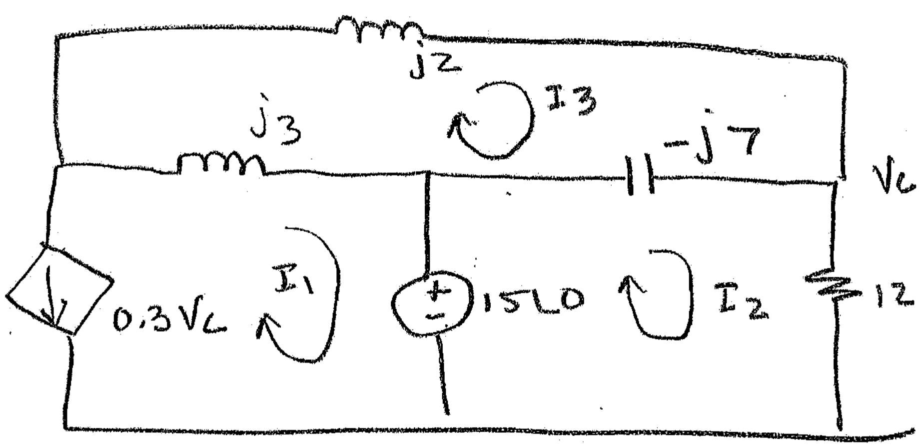
② $-\frac{1}{j2} V_a + \left[\frac{1}{12} + \frac{1}{-j7} + \frac{1}{j2}\right] V_c = -\frac{15}{j7}$

Solving gives

$V_a = 5.29 - j2.10$

$V_c = \underbrace{0.6746 - j3.11 = V_{TH}}$

Now we'll use mesh analysis and see if we get the same voltage for V_c.



(1) $I_1 = -0.3\,V_c = -0.3 \times 12\, I_2 \Rightarrow I_1 + 0.3 \times 12\, I_2 = 0$

(2) $-15 + (I_2 - I_3) \times -j7 + I_2 \times 12 = 0$

$-15 - j7 I_2 + j7 I_3 + 12 I_2 = 0$

$(-j7 + 12) I_2 + j7 I_3 = 15$

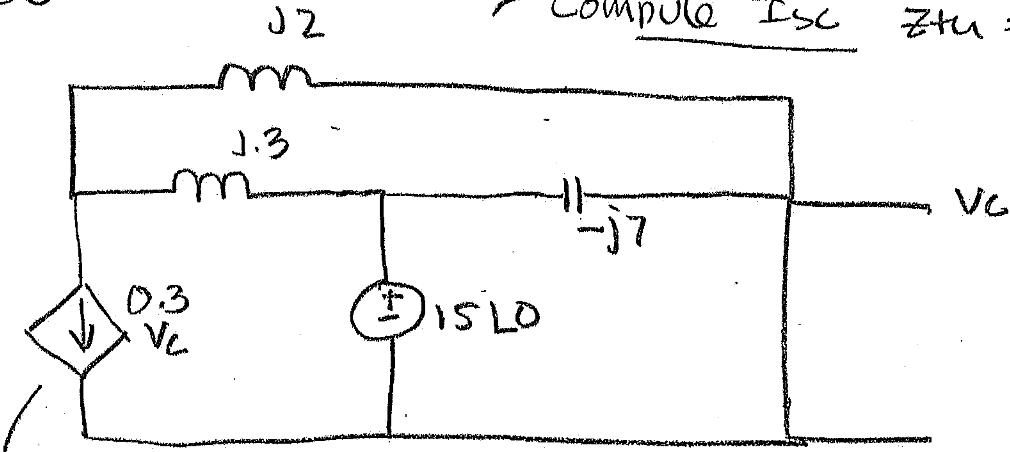
(3) $(I_3 - I_2) \times -j7 + (I_3 - I_1) j3 + I_3 j2 = 0$

$-j3\, I_1 + j7\, I_2 + (j3 - j7 + j2) = 0$

$$\begin{bmatrix} 1 & 0.3 \times 12 & 0 \\ 0 & -j7 + 12 & j7 \\ -j3 & j7 & j3 - j7 + j2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 15 \\ 0 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

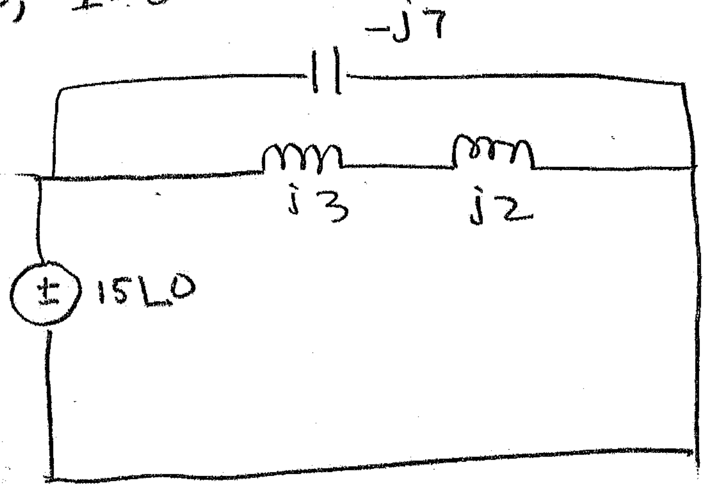
$\underbrace{12 I_2 = V_c = 0.6746 - j3.11 \text{ V}}_{\text{This agrees with mesh analyses}} = V_{Th}$

Get Z_{TH} $\longrightarrow$ Compute I_{sc} $\quad Z_{TH} = \dfrac{V_{TH}}{I_{sc}}$



Since $V_c = 0$, $I = 0$

Vc=0 so
Current source
is open



$$I_{sc} = \frac{15\angle 0}{-j7 \,//\, j5} = \frac{15}{\dfrac{35}{-j7+j5}} = \frac{15 \times -j2}{35} = \frac{-j30}{35}$$

$$= -j0.857$$

$$\text{So } \frac{V_{TH}}{I_{sc}} = Z_{TH} = \frac{0.6746 - j3.11}{-j0.857} = \underbrace{3.63 + j0.787\ \Omega}_{Z_{TH}}$$

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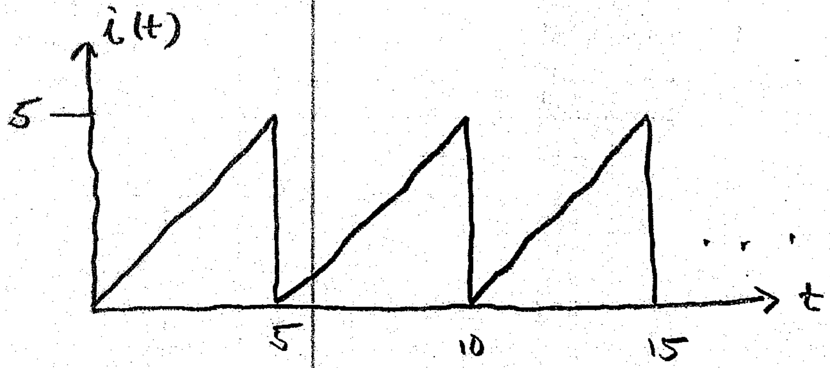

$i(t)$ is a line (over one period)

so $i(t) = mt + b$ and $m = 1$, $b = 0$

$$i_{rms} = \left[\frac{1}{T}\int_0^T t^2\,dt\right]^{1/2} = \left[\frac{1}{5}\left.\frac{t^3}{3}\right|_0^5\right]^{1/2}$$

$$= \left[\frac{1}{5} \times \frac{5^3}{3}\right]^{1/2} = \sqrt{\frac{125}{15}} = 2.887\,A$$