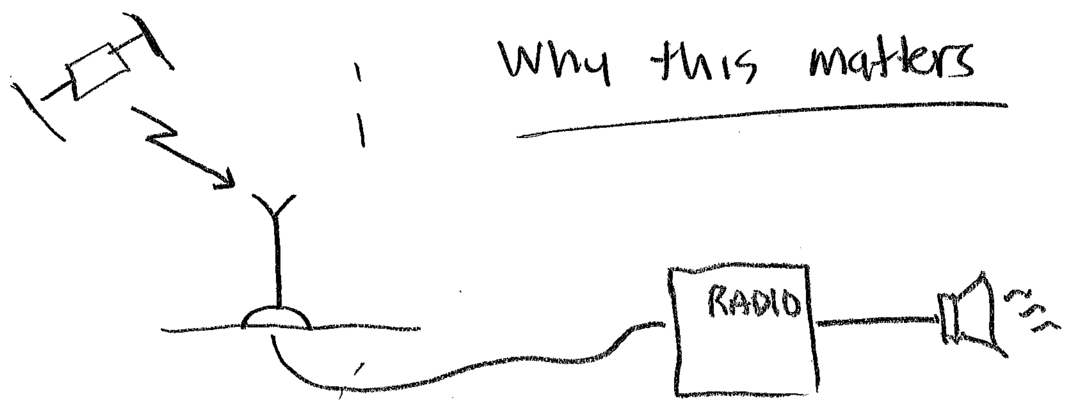
LECTURE 11

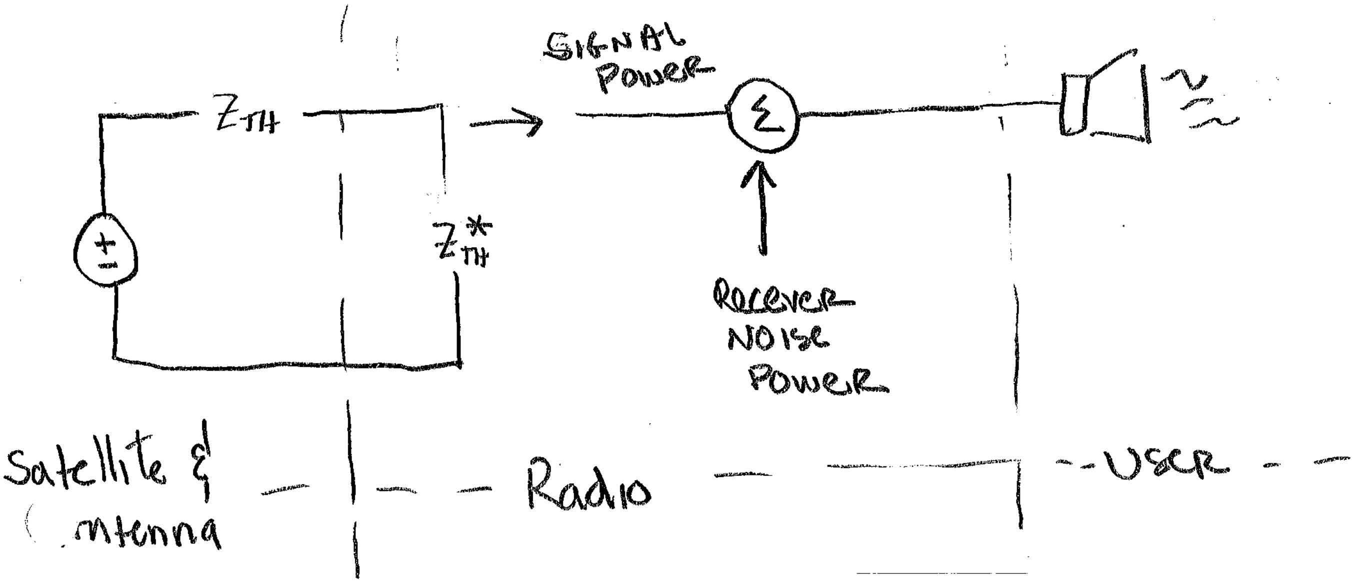
EFFECTIVE OR RMS VALUES

"For maximum average power transfer, the load impedance Z_L must be equal to the complex conjugate of the Thevenin impedance Z_{Th}"

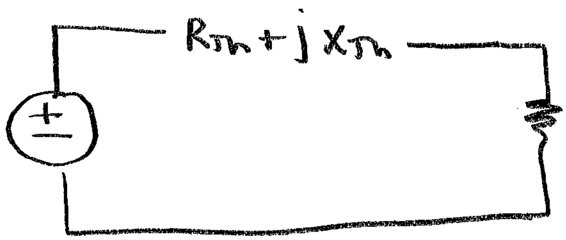
PG 465

Why this matters





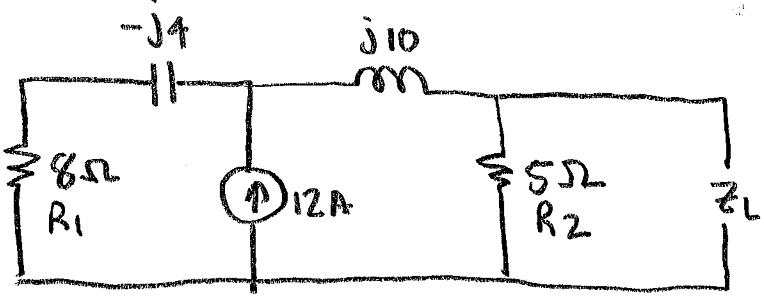
Say the Thevenin source is complex, but you can <u>only</u> add a resistor



$R_L = \sqrt{R_{TH}^2 + X_{TH}^2} = |Z_{TH}|$

EQ 11.2|

PRACTICE Problem 11.5 Pg 466



- Find Z_L for maximum Power transfer
- Find R_L for maximum Power transfer
- Find Load power in both cases

STRATEGY: Represent as thevenin source

$$Z_L = Z_{TH}^*$$

$$Z_L \text{ (resistor only)} = |Z_{TH}| \leftarrow \text{EQ 11.21}$$

$$\text{Pwr } (Z_L = Z_{TH}^*) = \frac{|V_{TH}|^2}{8R_{TH}} \leftarrow \text{EQ 11.20}$$

PwR (Resistor only) → Solve for current, use

$$P = \frac{1}{2}|I|^2 R$$

Thevenin Impedance

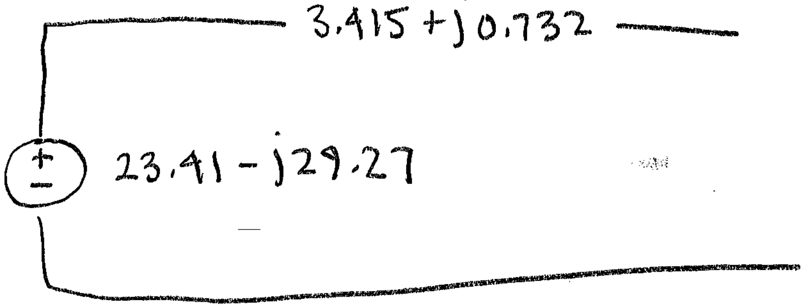
$$Z_{TH} = R_2 // (8 - j4 + j10) = R_2 // (8 + j6) = 5 // (8 + j6)$$

$$= \frac{5(8+j6)}{5+(8+j6)} = 3.415 + j0.732$$

Thevenin Voltage

$$\underbrace{12 \times \frac{(8-j4)}{8-j4+j10+5}}_{\text{Current in right branch}} \times 5 = 23.41 - j29.27 \text{ V}$$

× 5Ω



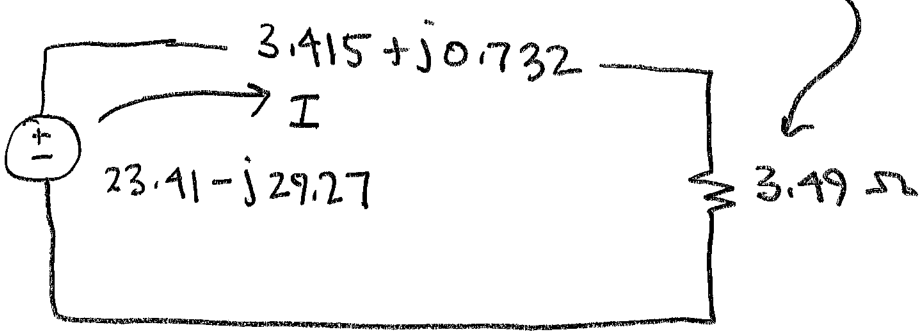
Theven representation for Prac Prob 11.5

So $Z_L = \underline{Z_{Th}^{*} = 3.415 - j0.732}$

$$P_{max} = \frac{|V_{Th}|^2}{8R_{TH}} = \frac{1405}{8 \times 3.415} = \underline{51.42\ W}$$

For max pwr in resistive load,

$$R_L = |Z_{TH}| = 3.49\ \Omega$$



$$I = \frac{23.41 - j29.47}{3.415 + j0.732 + 3.49} = 2.91 - j4.55$$

$$= 5.40 \angle -57.4$$

So $P = \frac{1}{2} \times |I|^2 R = \frac{1}{2} \times 5.40^2 \times 3.49 = \underline{50.88\ W}$

EQ 11.11

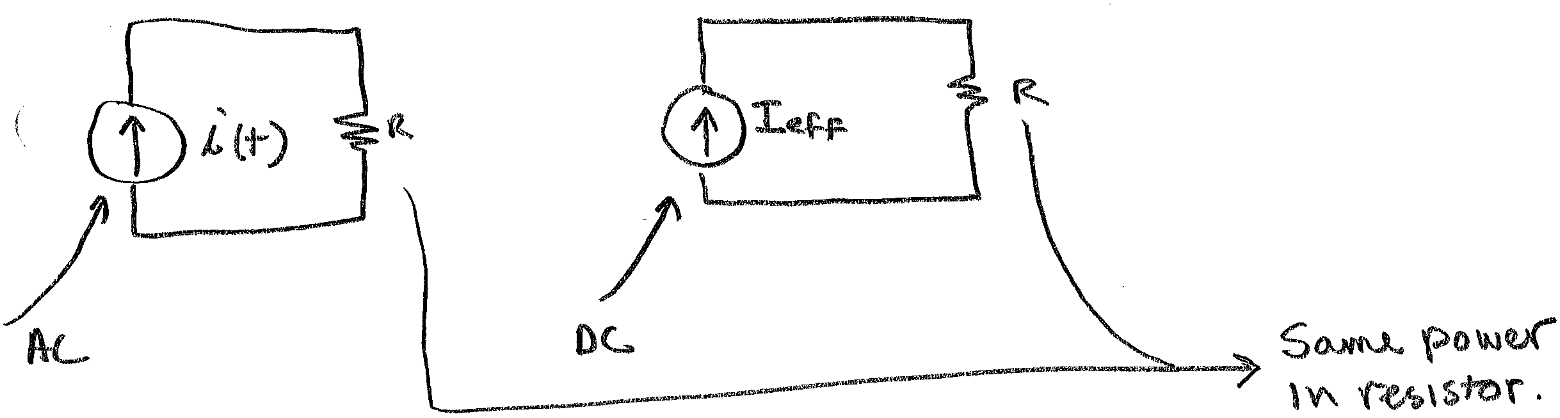
Root Mean Square (RMS) Sec 11.4

From EQ 11.11, $P_R = \frac{1}{2} V_m I_m$

½ is messy

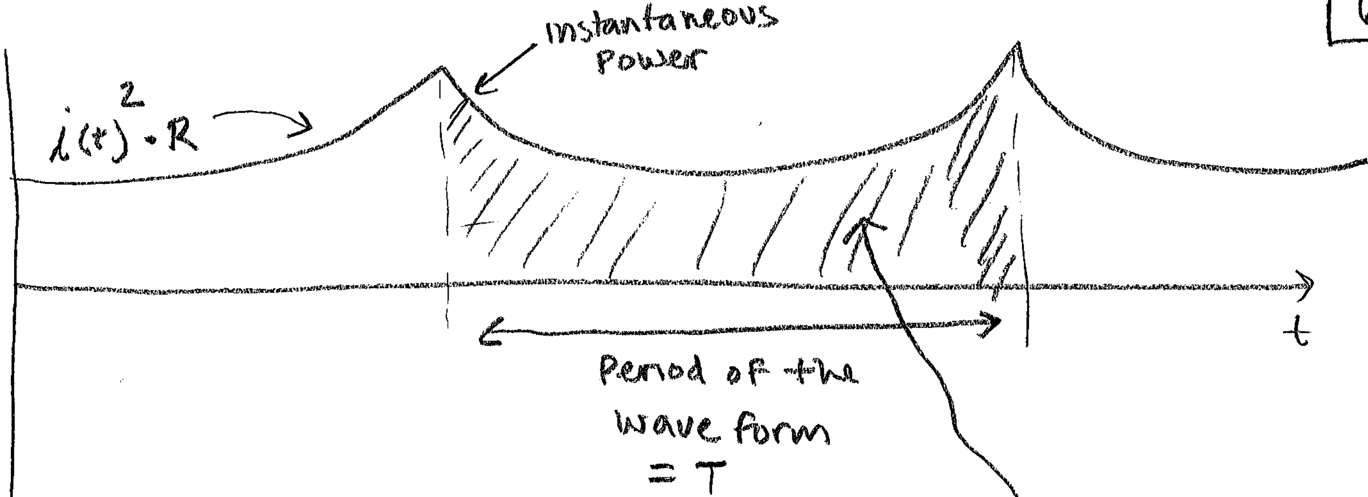
Here's how engineers like to describe voltages or currents:

" The <u>effective value</u> of a periodic current is the DC current that delivers the same average power to a resistor as the periodic current" (Page 467)



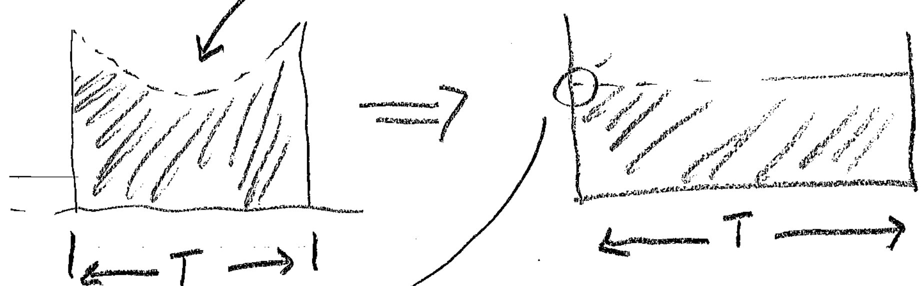
<u>Mathematically</u>

$$\text{Avg Power} = \frac{1}{T}\int_0^T i(t)^2 \cdot R \, dt = I_{eff}^2 \cdot R$$



$$\int_0^T i^2(t)\cdot R\, dt = \text{Area}$$

Energy per cycle



$$y = \frac{\text{area}}{T} = \frac{1}{T}\int_0^T i^2(t)\cdot R\, dt$$

= Average power

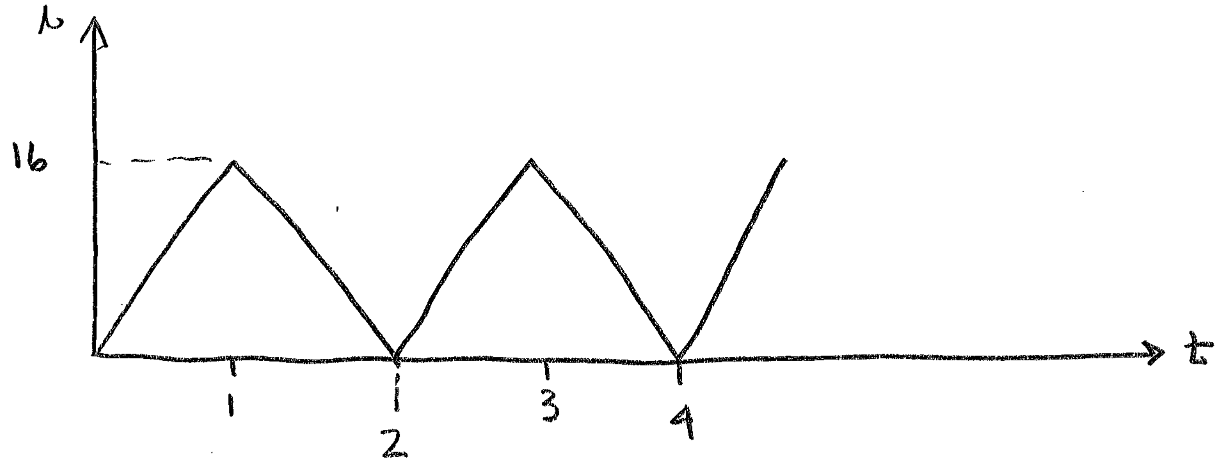
Now we relate AC and DC

$$\overbrace{I_{eff}^2 \cdot R}^{\text{Average power}} = \frac{R}{T}\int_0^T i^2(t)\, dt$$

So $I_{eff} = \sqrt{\underbrace{\frac{1}{T}\int_0^T i^2(t)\, dt}}$ → Root Of The Mean Square

PRACTICE problem 11.7 Pg 469

...nd the rms value of the current. If this current flows through a 9 Ω resistor, calculate the average power.



$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

slope of line is 16 A/s so $i = 16t$ $(0 \le t \le 1)$

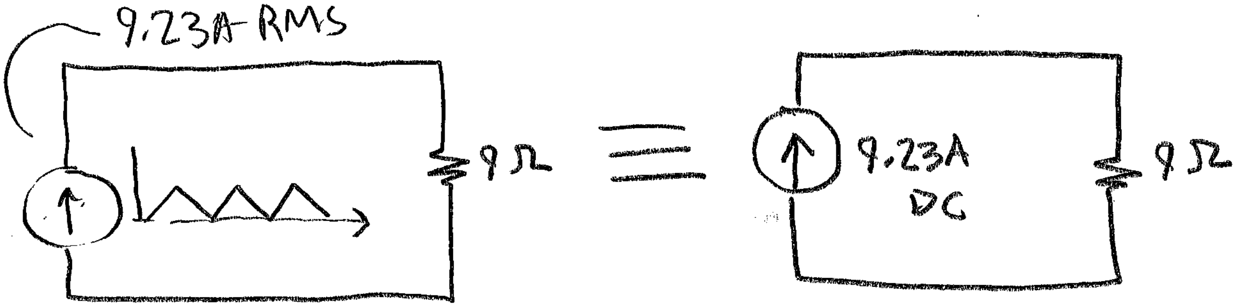
so $i^2(t) = 256t^2$

By inspection, the integral from 1 to 2 = integral from 0 to 1

so $I_{eff} = \sqrt{\underbrace{\frac{1}{2}}_{\text{Period}} \times 2\int_0^1 256\,t^2\,dt}$

double the integral and integrate from 0 to 1 instead of 0, 2

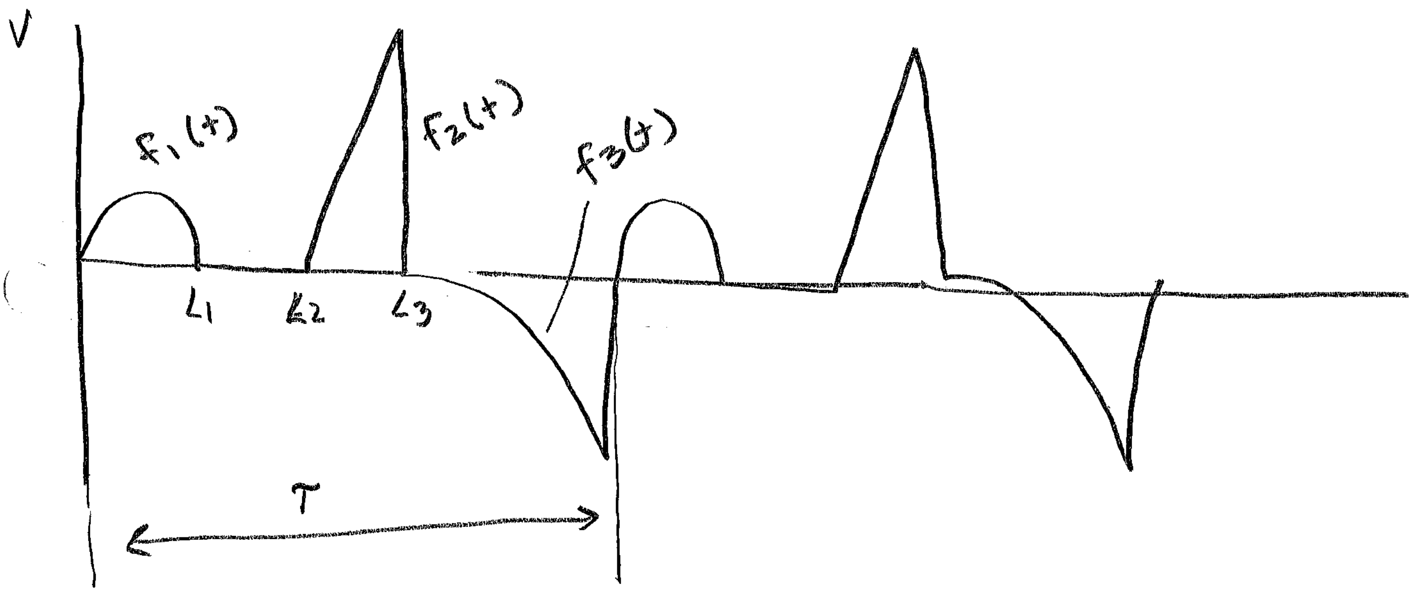
$$I_{eff} = \sqrt{256 \times \left.\frac{t^3}{3}\right|_0^1} = \sqrt{256/3} = \underline{9.23\,A}$$



$I_{RMS}^2 \times R = 9.23^2 \times 9 = 768\text{ W}$

$I^2 R = 9.23^2 \times 9 = 768\text{ W}$

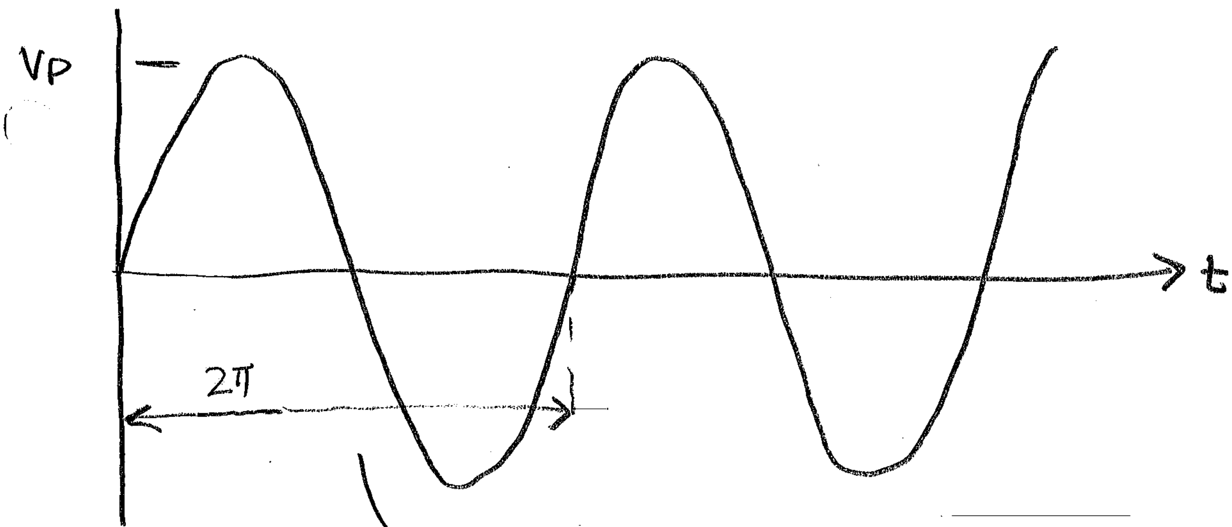
Example shows that if we can integrate the periodic waveform we can compute the RMS value.



$$V_{rms} = \sqrt{\frac{1}{T}\left[\int_0^{L_1} f_1^2(t)\,dt + \int_{L_2}^{L_3} f_2^2(t)\,dt + \int_{L_3}^{T} f_3^2(t)\,dt\right]}$$

∴ even this awful waveform has an rms value

Note RMS depends on shape and amplitude only



Express period using phase

Sinusoidal voltage (or current)

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_p^2 \sin^2(\phi)\, d\phi}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_p^2 \cdot \frac{1}{2}(1-\cos(2\phi))\, d\phi}$$

Zero by inspection

$$V_{rms} = V_p\sqrt{\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}\, d\phi - \cancel{\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}\cos(2\phi)\, d\phi}}$$

or integrate!

$$V_{rms} = V_p\sqrt{\frac{1}{2\pi}\times\frac{1}{2}\times 2\pi - \cancel{\frac{1}{2\pi}\cdot\frac{1}{2}\cdot\Big|_0^{2\pi}\frac{1}{2}\cdot\sin(2\phi)}^{\,0}}$$

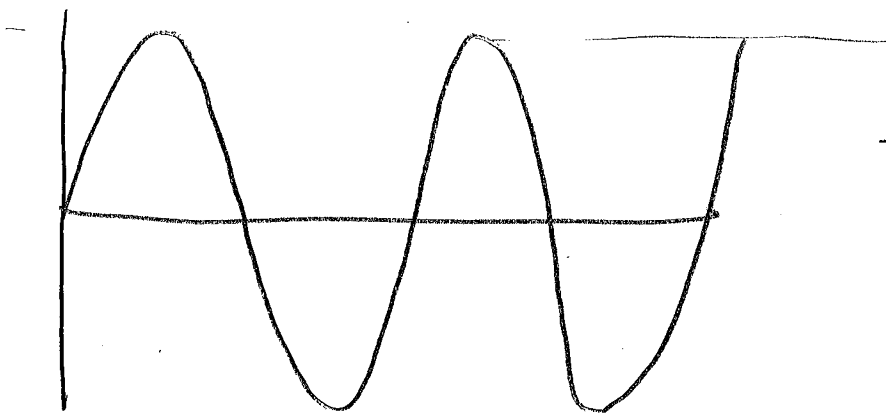
$$\boxed{V_{rms} = V_p \times \frac{1}{\sqrt{2}}}$$

EQ 11.29

When we describe household voltage, it's 120 VAC, which is an RMS value

$$V_{rms} = \frac{1}{\sqrt{2}} \times V_{pk} = 0.707 \times pk$$

So $V_{pk} = \sqrt{2}\, V_{rms} = \sqrt{2} \times 120 = 169.7\ V_{pk}$



$V_{pk} = 169.7\ V$

Remember $P_R = \frac{1}{2} V_m \cdot I_m$ for a resistor?

$V_m = \sqrt{2} \times V_{rms}$, $I_m = \sqrt{2}\, I_{rms}$

So

$$P_R = \frac{1}{2} \sqrt{2}\, V_{rms} \times \sqrt{2}\, I_{rms} = V_{rms} \times I_{rms}$$

This holds because

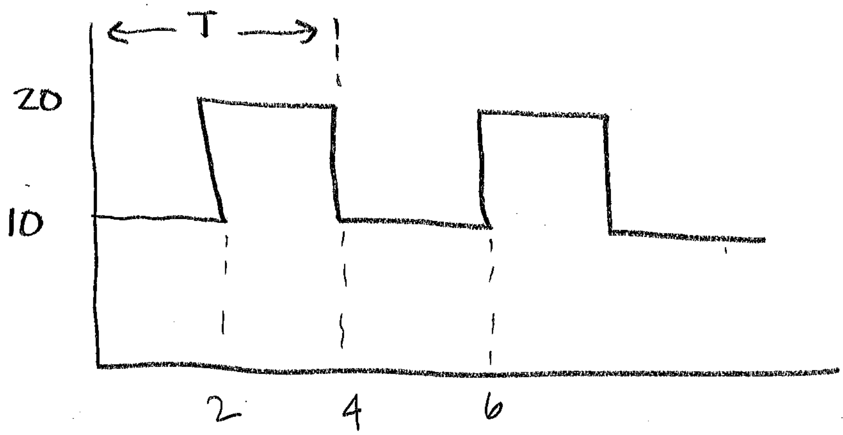
$P_R = \frac{1}{2} V_m \cdot I_m \longrightarrow$ EQ 11.11

and $P = V_{rms} \times I_{rms} \longrightarrow$ EQ 11.30 with $\cos\theta = 1$

were developed for sinusoidal signals.

PR 11.26 Pg 493

ind the effective value of the signal.
Find the peak value of a sinusoidal signal that will deliver the same power to a resistor.



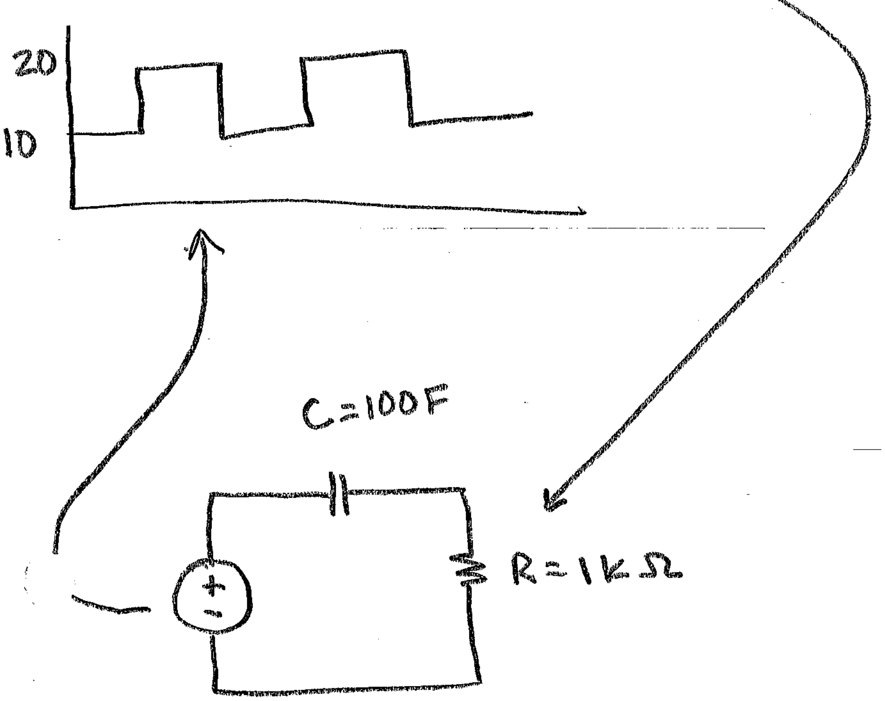
$$V_{rms} = \sqrt{\frac{1}{4}\int_0^2 10^2 dt + \frac{1}{4}\int_2^4 20^2 dt}$$

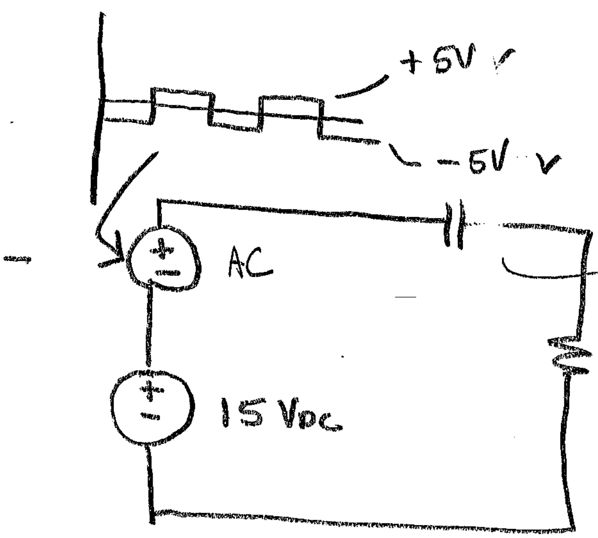
$$V_{rms} = \sqrt{\frac{1}{4}\Big|_0^2 100t + \frac{1}{4}\Big|_2^4 400t}$$

$$= \sqrt{\frac{1}{4}\left[200 + (400 \times 4 - 400 \times 2)\right]} = 15.811\,V_{rms}$$

b) Since $V_{rms} = 15.811$ V, V_{pk} for a sinewave with this rms value is $15.811 \times \sqrt{2} = 22.36$ V

COMPUTE V_{RMS} Here





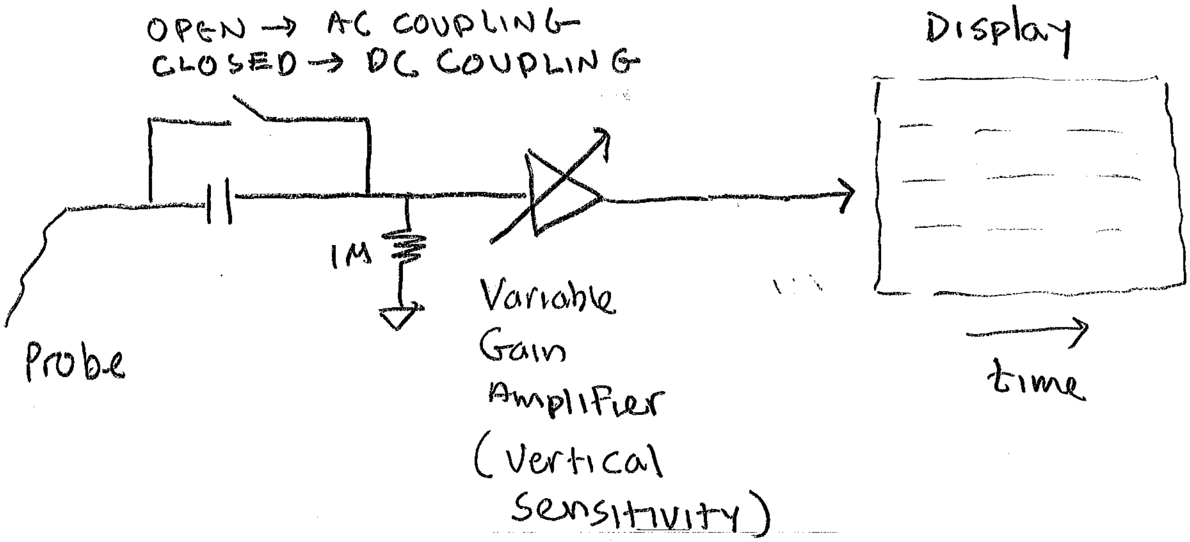
Split Source

$$\frac{1}{j\omega C} \stackrel{?}{=}$$

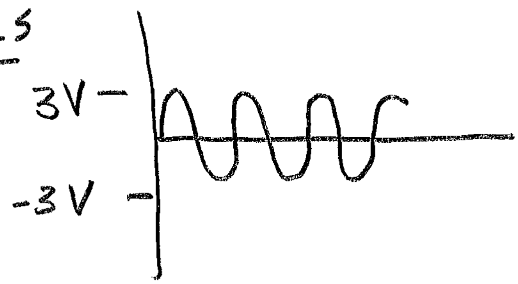
$$\sqrt{\frac{1}{T}\int_0^T 5^2\,dt}$$

$$= \sqrt{\frac{T}{T}25} = 5\text{ V}$$

OSCILLOSCOPE COUPLING

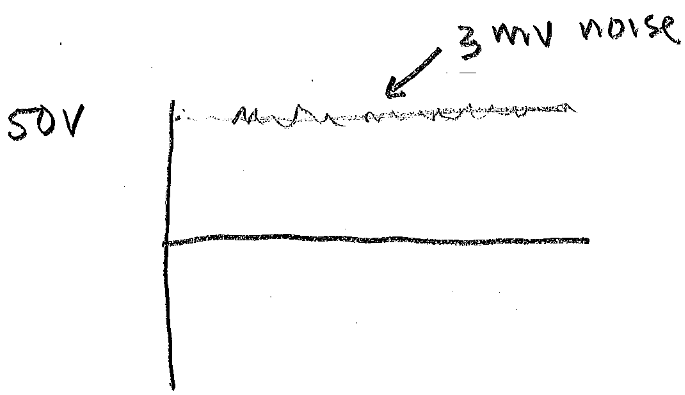


SIGNALS



What Scale?

What Coupling?



What Scale?

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