# LECTURE 10 - INSTANTANEOUS AND AVERAGE POWER

## BACKGROUND CONCEPTS:

ENERGY: "I hiked up the mountain" (JOULES)

POWER: "I can hike 1000 vertical feet in one hour" (Joules/second = Watts)

work and power: "I hiked up the mountain in an hour"

Your Power bill: KW-Hrs $\longrightarrow$ $\frac{J}{S} \times S$ = Joules

Useful relationship: 1 Horsepower = 746 Watts.

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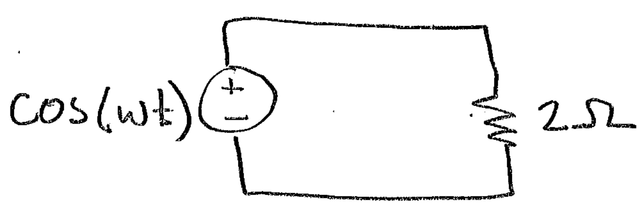
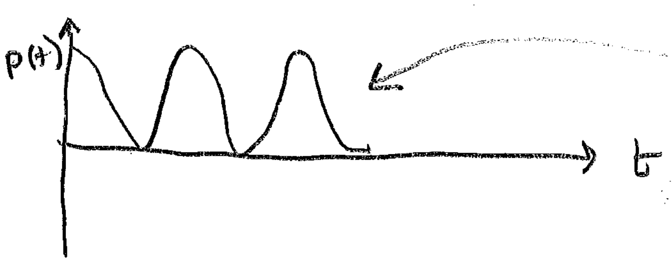
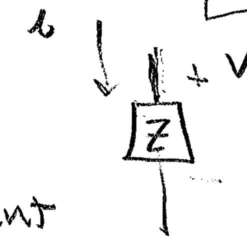
## Instantaneous Power

$$P(t) = V(t)\, i(t) \quad \text{EQ 11.1}$$




$$P(t) = V(t)\, i(t) = \cos(\omega t) \times \frac{\cos(\omega t)}{2} = \frac{1}{2}\cos^2(\omega t)$$






$v(t) = V_m \cos(\omega t + \phi_v)$ — Instantaneous voltage

$i(t) = I_m \cos(\omega t + \phi_i)$ — Instantaneous current

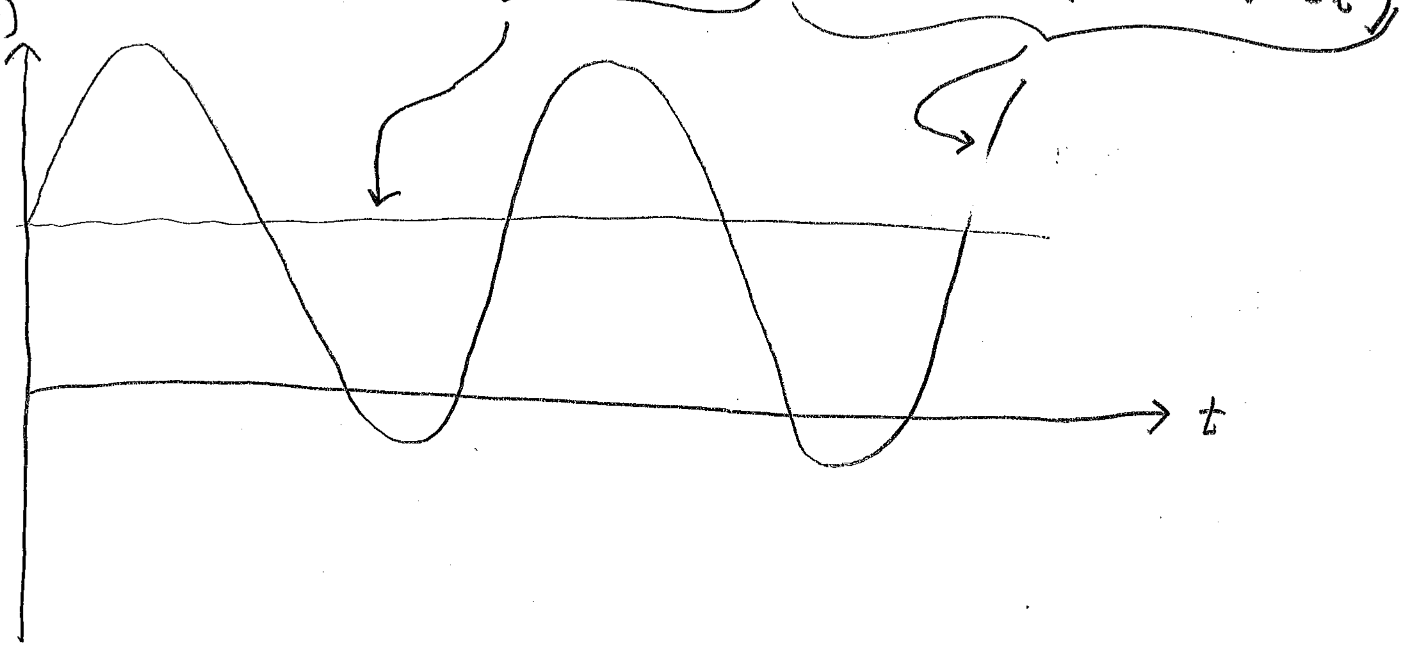
Instantaneous Power is

$$P(t) = v(t)\, i(t) = V_m \cos(\omega t + \phi_v) \cdot I_m \cos(\omega t + \phi_i)$$

Use the trigonometric identity:

$$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

So $$P(t) = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \phi_v + \phi_i)$$



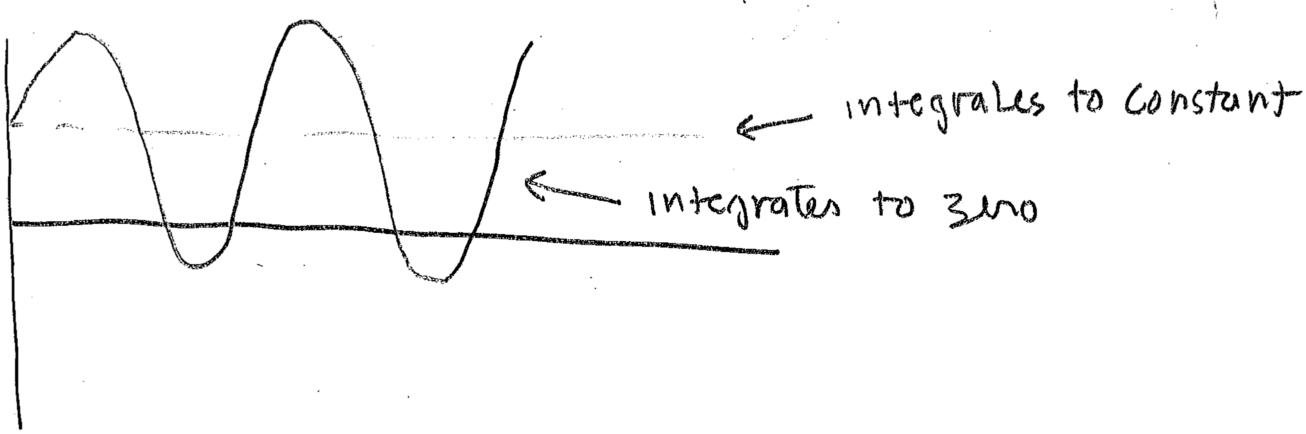

So instantaneous power has a constant part and a time varying part.

Average power is the constant part:

$$P = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i)$$

Definition of average power for a waveform with period T

$$P = \frac{1}{T}\int_0^T p(t)\,dt$$




Say v(t) and i(t) are expressed as phasors:

complex conjugate

$$\frac{1}{2} V \cdot I^* = \frac{1}{2} V_m I_m \angle \phi_v - \phi_i$$

$$= \underbrace{\frac{1}{2} V_m I_m \cos(\phi_v - \phi_i)}_{\text{This term is the average power!}} + j\,\frac{1}{2} V_m I_m \sin(\phi_v - \phi_i)$$

So $$P = \frac{1}{2}\mathrm{Re}\{V \times I^*\} = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i)$$ EQ 11.10

# Power in Components

**Resistor** – voltage and current are in phase.

$$P = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i) = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 \cdot R$$

Why isn't power = V × I like it is for DC ??

**INDUCTOR**

$$\frac{V}{I} = j\omega L \qquad \text{so} \qquad V = j\omega L I$$

Voltage and current are 90° out of phase

$$P = \frac{1}{2} V_m I_m \cos(\cancel{\phi_v - \phi_i}) = 0$$

Inductor is an energy storage element

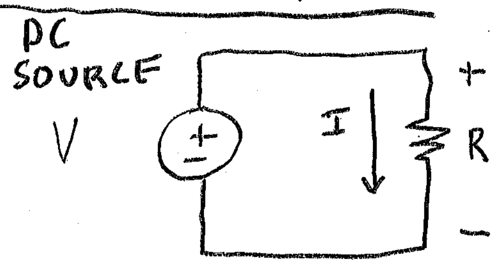
**Capacitor**

$$\frac{V}{I} = \frac{1}{j\omega C} \quad \text{or} \quad V = I \times \frac{1}{j\omega C} = -I \times \frac{1}{\omega C} \times j$$

$$P = \frac{1}{2} V_m I_m \cos(\cancel{\phi_v - \phi_i}) = 0$$

Voltage and current are out of phase

# Power Supply




Passive Sign Convention

Power dissipated by resistor is $V \times I$ (Positive)

Power dissipated by supply is $V \times (-I) = -VI$ (Negative)

The power supply dissipates negative power.

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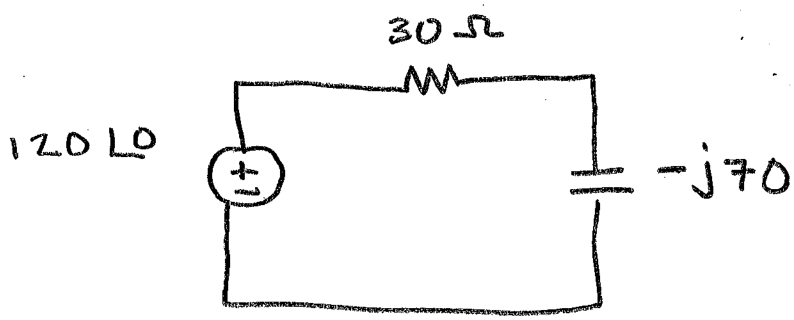
EXAMPLE:

Compute power dissipated in the circuit




$$I = \frac{V}{Z} = \frac{120 + j0}{30 - j70} = 1.58 \angle 66.8^\circ \text{ A}$$

$$V_R = I_R \times R \doteq 1.58 \angle 66.8^\circ \times 30 \text{ V}$$

CONJ

$$P = \frac{1}{2} Re\,(V \times I^*) = \frac{1}{2} Re \left\{ 1.58 \angle 66.8^\circ \times 30 \times 1.58 \angle -66.8^\circ \right\} \text{W}$$

$$= \frac{1}{2} Re \left\{ 74.89 \angle 0^\circ \right\} = \underline{37.44 \text{ W}}$$

What is Supply Power?

How many joules are delivered to the resistor in 10 seconds?

If the resistor were shaped like a golf ball and you held it in your hand, would you get burned?

Practical power formula

In the previous example, we know only the resistor can dissipate power.
We also know voltage and current for the resistor are in phase.

so $$P = \frac{1}{2} V_m I_m \overbrace{\cos(\Delta\phi)}^{1.0} = \frac{1}{2} V_m I_m$$

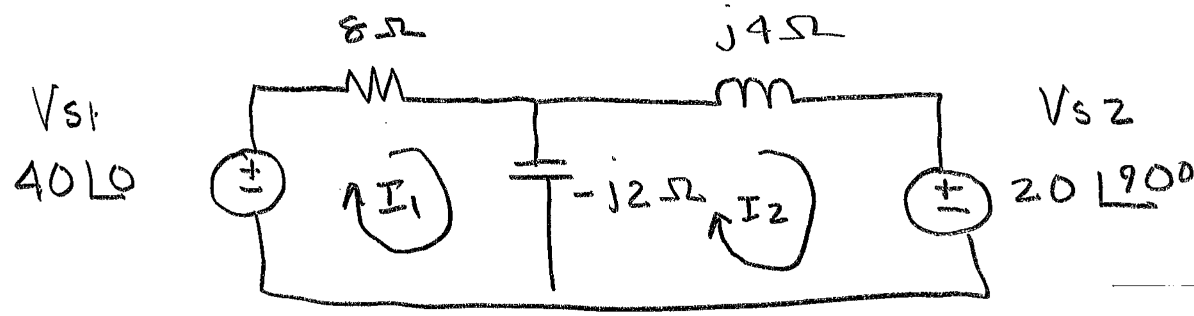
$V_m = I_m \cdot R$ so $P = \frac{1}{2} I_m \cdot R \times I_m$

or $$\boxed{P = \frac{1}{2} I_m^2 \times R = \frac{1}{2} |I|^2 \cdot R}$$ EQ 11.10

In the last example, $|I| = 1.58\,A$ so $P = \frac{1}{2} \times 1.58^2 \times 30$

$= 37.4\,W$

Calculate the power absorbed by each of the five elements in the circuit.




Triage the problem

Capacitor power = 0
Inductor power = 0
Resistor power is positive

$P_{S1} + P_{S2} + P_R = 0$ ⟶ "Power from Source = pwr diss" in Resistor

$P_{S1}$, $P_{S2}$ ⟶ Negative power dissipation

If I can get the two mesh currents $I_1, I_2$ then

$P_R = \frac{1}{2}|I_1|^2 \cdot R$ ⟶ Power-dissipated by source 1

$P_{VS1} = -\frac{1}{2}|V_{S1}| \times |I_1| \cos(\phi_{V1} - \phi_{I_1})$ Neg because $I_1$ leaves $V_{S1}$

$P_{VS2} = \frac{1}{2}|V_{S2}| \times |I_2| \cos(\phi_{VS2} - \phi_{I2})$ Pos because $I_2$ enters $V_{S2}$

Get $I_1, I_2$ 

(1) $-V_{s1} + 8I_1 - j2(I_1 - I_2) = 0$

$(8 - j2)I_1 + j2\,I_2 = V_{s1}$

(2) $-j2(I_2 - I_1) + j4\,I_2 + V_{s2} = 0$

$j2\,I_1 + (j4 - j2)I_2 = -V_{s2}$

so
$$\begin{bmatrix} 8-j2 & j2 \\ j2 & j4-j2 \end{bmatrix}^{-1} \begin{bmatrix} V_{s1} \\ -V_{s2} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$V_{s1} = 40 + j0 \qquad V_{s2} = j20$

$I_1 = 3 + j4 \qquad I_2 = -13 - j4$

$I_1 = 5\angle 53.13^\circ \qquad I_2 = 13.6\angle -162.9$

$= 0$

$P_R = \frac{1}{2}|I_1|^2 \times 8 = 100W$ +

$P(\text{source 1}) = \frac{1}{2}|V_{s1}| \times |I_1| \cos(0 - \phi_{I_1})$

$= -\frac{1}{2} \times 40 \times 5 \times \cos(-53.13^\circ) = -60W$

+

$P_{\text{source 2}} = \frac{1}{2}|V_{s2}| \times |I_{s2}| \times \cos(90 + 162.9^\circ)$ check

$= \frac{1}{2} \times 20 \times 13.6 \times \cos(90 + 162.9) = -40\,W$

# Maximum Power Transfer



Consider the Thevenin Source

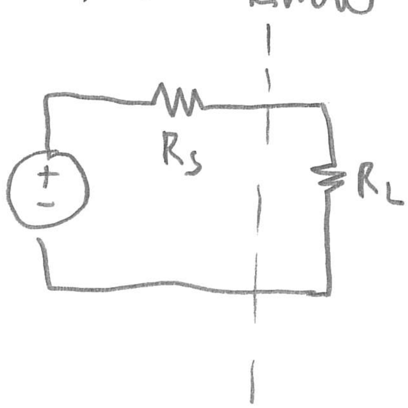

Load the source with an inductor
Load the source with a capacitor
Load the source with an open circuit
Load the source with a short circuit

How much power dissipated in load?

What is needed to dissipate power??

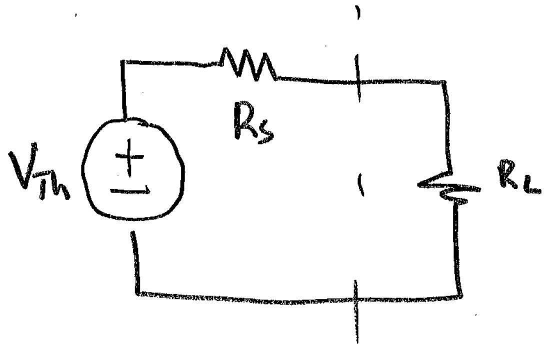
Big resistor → No power because minimal current
Small resistor → No power because minimal voltage.

For DC, we know



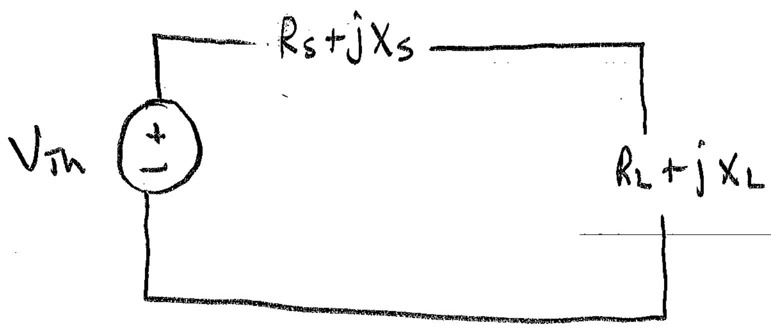

For a source with Thevenin resistance $R_s$, max power transfer occurs when
$R_L = R_s$

## Tricky interview question




Pick $R_s$ for maximum power transfer

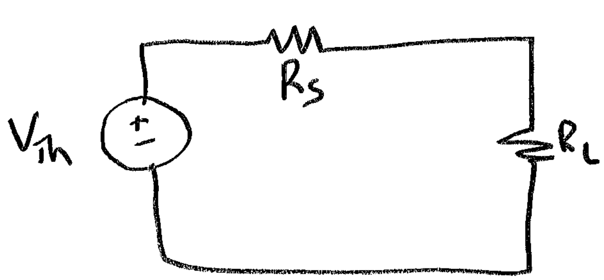
## Maximum power transfer for AC circuits




See derivation on Pg 465 of Text.

## Intuitive 'derivation'

- Load power will be dissipated in $R_L$ only.
- If $jX_s + jX_L = 0$ then the circuit becomes




and for max power transfer, $R_L = R_s$

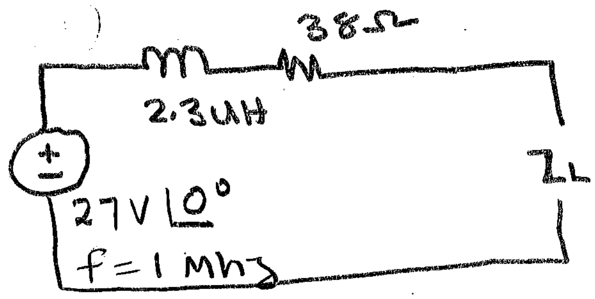
- If $jX_s + jX_L \neq 0$ current will decrease and $|I|^2 R_L$ will decrease also

IF $jX_S + jX_L = 0$ then $jX_S = -jX_L$

or $Z_L = Z_S^*$

Example

Find $Z_L$ for maximum power transfer.
Find maximum power delivered to load.




$$Z_S = 38 + Z_{IND}$$

$$= 38 + j2\pi \times 1E6 \times 2.3uH$$

$$= 38 + j14.45\,\Omega$$

so $Z_L = Z_S^* = 38 - j14.45$

$\downarrow$

Negative so we need a capacitor

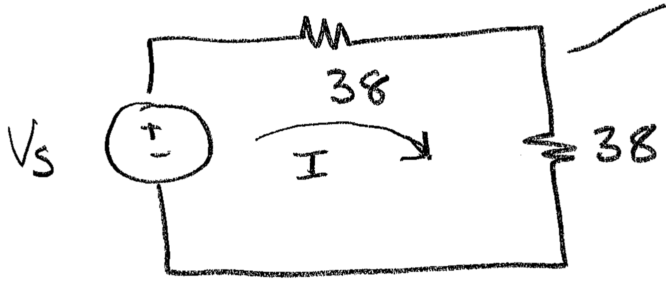
$$-j14.45 = \frac{1}{j\omega C}$$

$$C = \frac{1}{-j^2 \times 14.45 \times 2\pi \times 1MHz} = 11.01nF$$

$\longrightarrow$

Effective Circuit

Reactive parts canceled




How much power at max power transfer ?

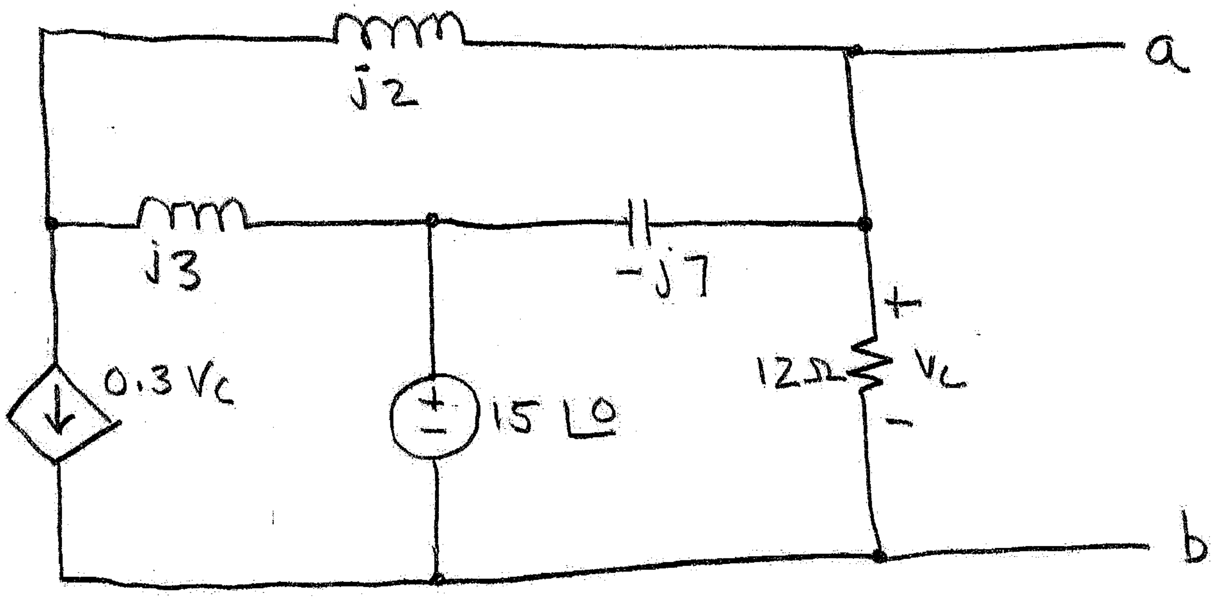
$$I = \frac{V_s}{2R_L} \qquad P_{max} = \frac{1}{2}|I|^2 \times R_L$$

$$= \frac{1}{2}\,\frac{V_s^2}{4R_L^2} \times R_L = \frac{V_s^2}{8R_L} \leftarrow \text{EQ } 11.20$$

For our example, this is

$$\frac{V_s^2}{8R_L} = \frac{27^2}{8 \times 38} = \underline{2.40\,W}$$

# Supplement for ASSIGN #5




1) Determine the load that can be placed between terminals a and b that will receive maximum power transfer from the circuit.

2) Determine the power delivered to this load.

Hints: 1) Use KVL or KCL to get $V_{TH}$

2) For Thevenin impedance, see "case 2" on pg 140

3) I prefer getting $Z_{TH}$ by computing $I_{SC}$ then

$$Z_{Th} = \frac{V_{TH}}{I_{SC}}$$